CHAIN CODE FOR MULTI-RESOLUTION ENCODING OF RASTERIZED 2D GEOMETRIC OBJECTS

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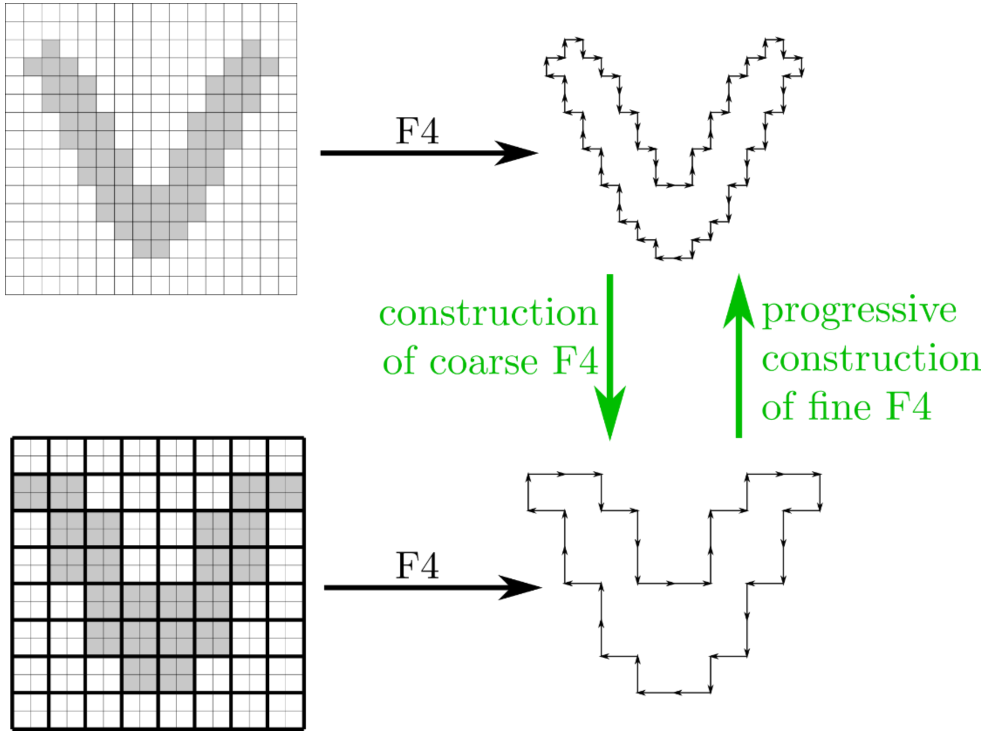
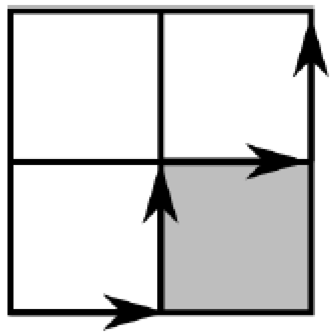
Motivation

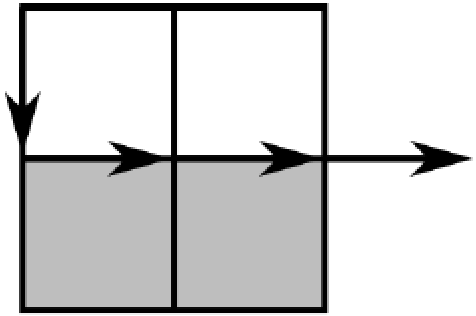
- Chain code -> efficient description of 2d geometric objects;
- Can be used for presentation or for determining object/shape properties;



Basic idea

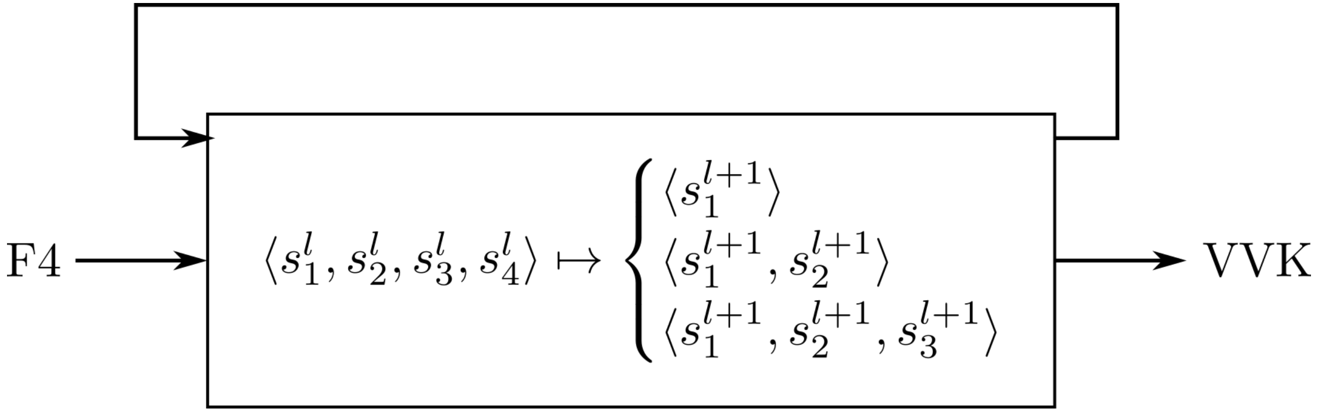
- Crack F4 chain code, **which starts at coarser grid point**, in a 2x2 cell [Chen & Chen, 2001]:
  - Four F4 moves reach coarser grid point;
  - Four F4 moves leave the coarser grid cell;
- Building block is a quadruple $Q^A = \langle s_0^A, s_1^A, s_2^A, s_3^A \rangle$;



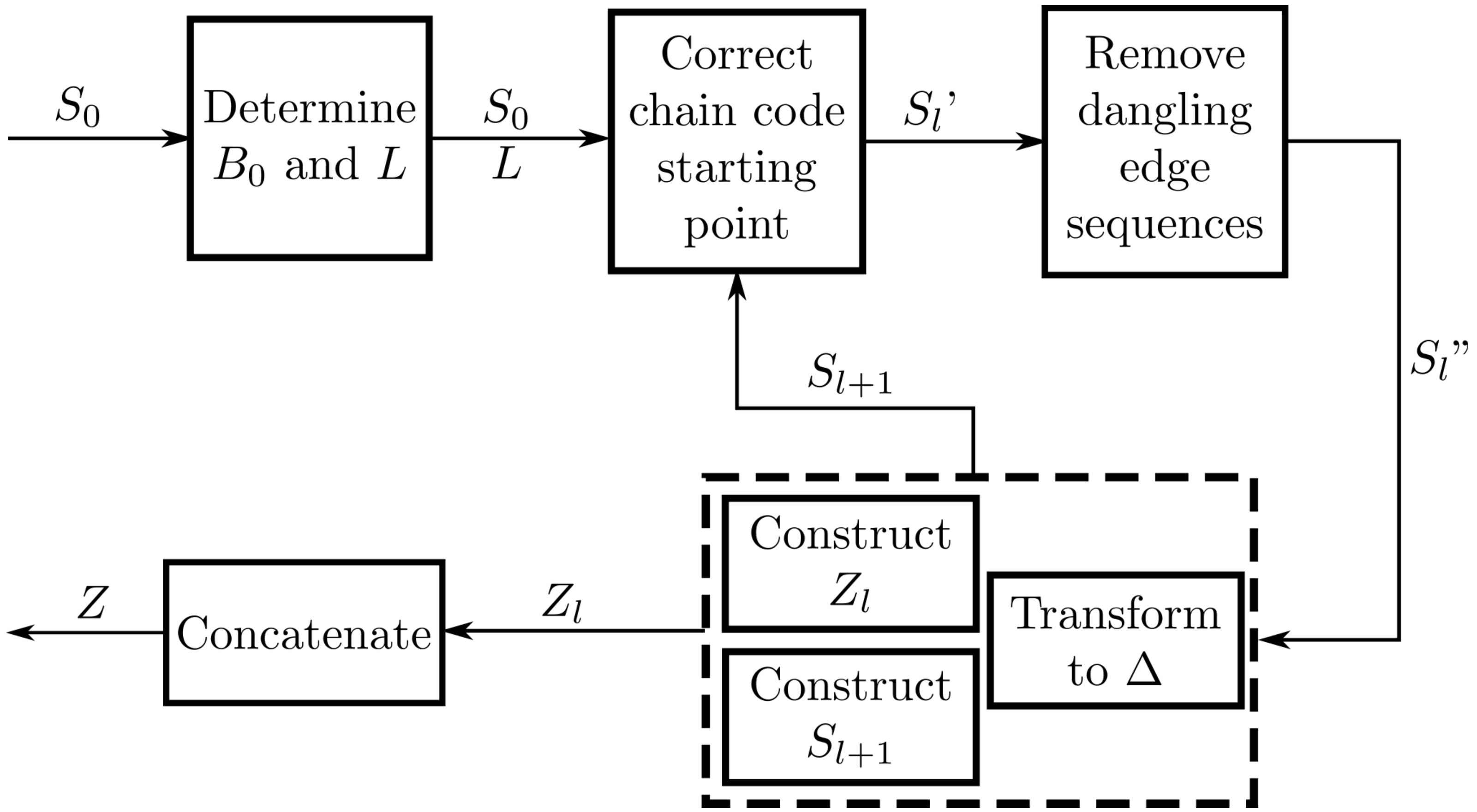


Basic goals

- Input: crack Freeman chain code in four directions (S_0);
- Map at most four symbols from input chain code into at most three symbols in the coarse F4 chain code;
- All combinations of four symbols of F4 alphabet => base alphabet for new multiresolution chain code (VVK, MrCC);
- Reducing 256 symbols of new alphabet to at most 32 symbols;

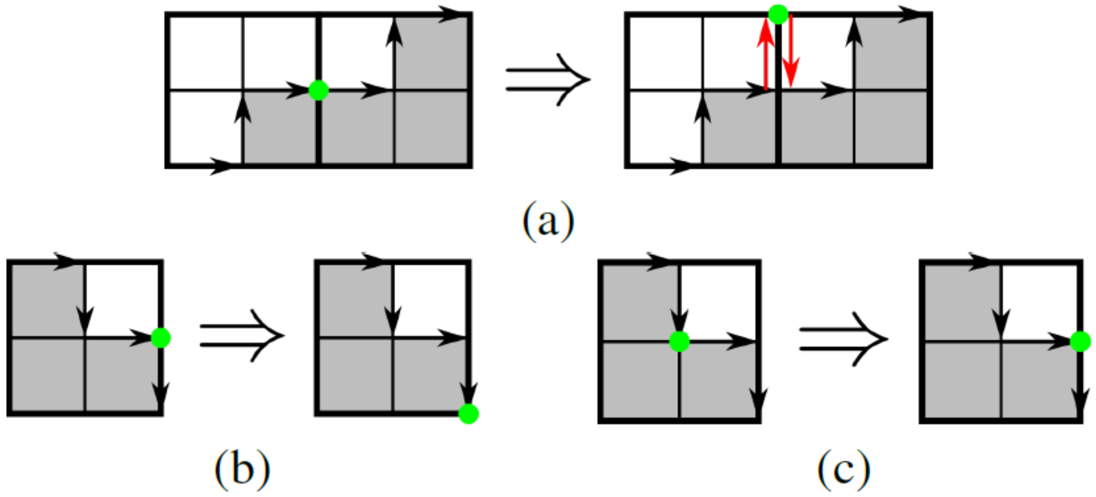


Solution overview

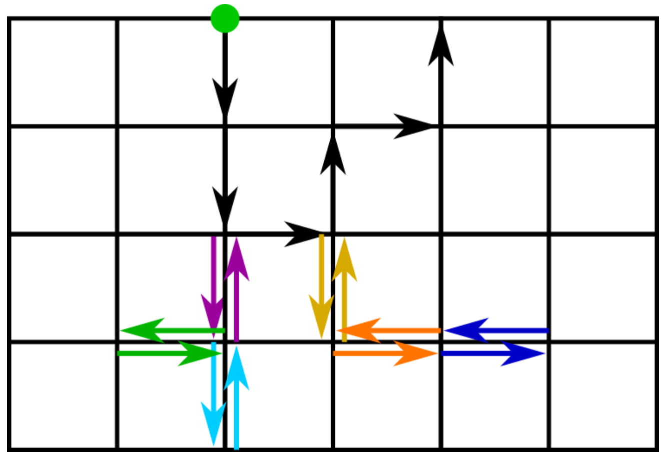


Solution overview

- Correct the starting point



- Remove all dangling edge sequences



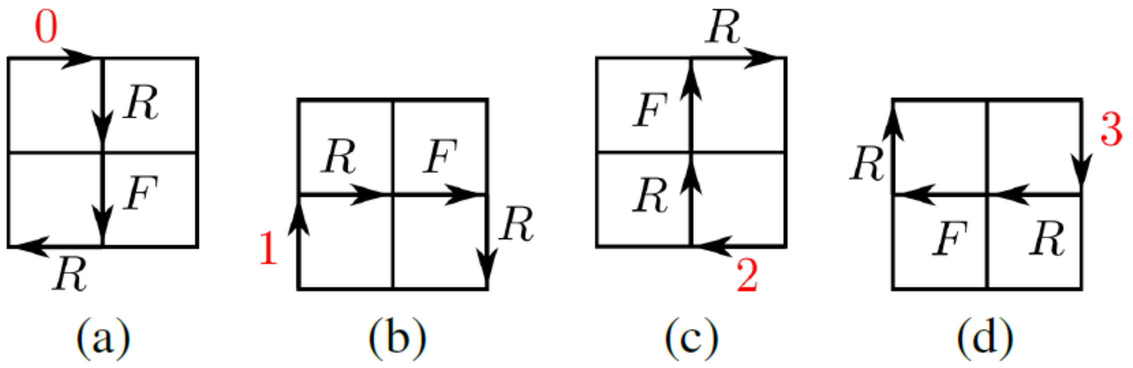
Lookup table

- 256 possible sequences of four F4 symbols $\langle s_0^A, s_1^A, s_2^A, s_3^A \rangle$;
- Combinations with dangling edges can be removed;
- Grid point reached after two moves: $s_0^A = s_1^A$;
- Rotations;

$$\Delta(Q) = \langle s_0^A, \delta_q \rangle, \quad q = \{1, 2, 3\}$$

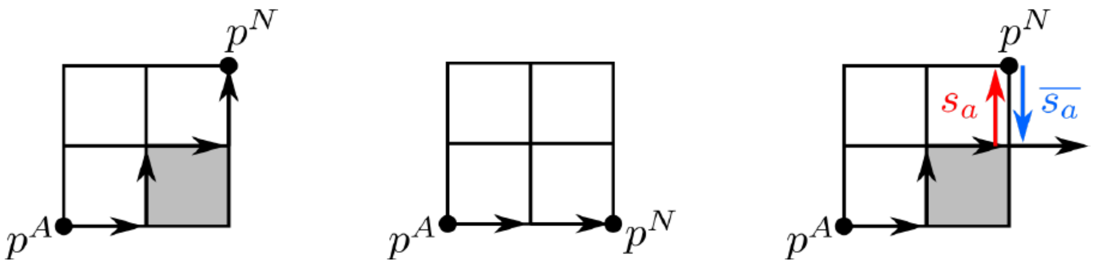
where

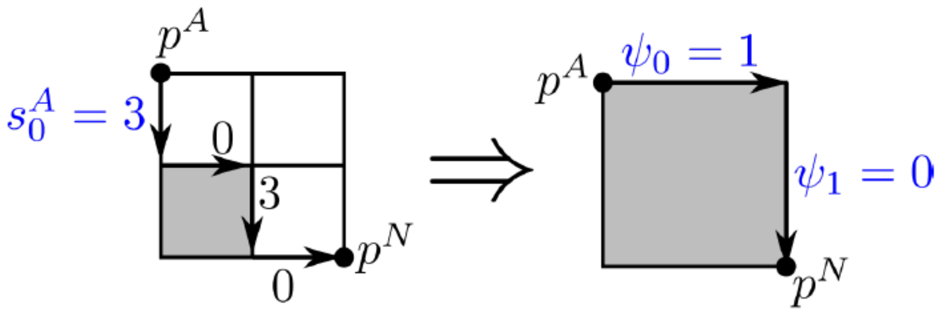
$$\delta_q = \begin{cases} F; & \textbf{if } s_q^A = s_{q-1}^A \\ L; & \textbf{if } s_q^A = (s_{q-1}^A + 1) \textbf{ mod } 4 \\ R; & \textbf{if } s_q^A = (s_{q-1}^A + 3) \textbf{ mod } 4 \end{cases}$$



Lookup table

$\langle \delta_1, \delta_2, \delta_3 \rangle$	MrCC	Ψ_{CW}	Ψ_{CCW}	$A = \langle \overline{s_a}, s_3^A \rangle$
$\langle F, \delta_2, \delta_3 \rangle$	A	$\langle 0 \rangle$	$\langle 0 \rangle$	
$\langle L, R, F \rangle$	B	$\langle 1, 0 \rangle$	$\langle 0 \rangle$	✓
$\langle L, R, L \rangle$	C	$\langle 1, 0 \rangle$	$\langle 0, 1 \rangle$	
$\langle L, R, R \rangle$	D	$\langle 1, 0, 3 \rangle$	$\langle 0 \rangle$	
$\langle L, F, R \rangle$	E	$\langle 1, 0 \rangle$	$\langle 0, 1 \rangle$	
$\langle L, F, F \rangle$	F	$\langle 1 \rangle$	$\langle 0, 1 \rangle$	✓
$\langle L, F, L \rangle$	G	$\langle 1 \rangle$	$\langle 0, 1, 2 \rangle$	
$\langle L, L, R \rangle$	H	$\langle 1 \rangle$	$\langle 0, 1, 2 \rangle$	
$\langle L, L, F \rangle$	I	$\langle \rangle$	$\langle 0, 1, 2 \rangle$	✓
$\langle L, L, L \rangle$	J	$\langle \rangle$	$\langle \rangle$	
$\langle R, L, F \rangle$	L	$\langle 0 \rangle$	$\langle 3, 0 \rangle$	✓
$\langle R, L, L \rangle$	M	$\langle 0 \rangle$	$\langle 3, 0, 1 \rangle$	
$\langle R, L, R \rangle$	N	$\langle 0, 3 \rangle$	$\langle 3, 0 \rangle$	
$\langle R, R, R \rangle$	O	$\langle \rangle$	$\langle \rangle$	
$\langle R, R, F \rangle$	P	$\langle 0, 3, 2 \rangle$	$\langle \rangle$	✓
$\langle R, R, L \rangle$	Q	$\langle 0, 3, 2 \rangle$	$\langle 3 \rangle$	
$\langle R, F, L \rangle$	R	$\langle 0, 3 \rangle$	$\langle 3, 0 \rangle$	
$\langle R, F, R \rangle$	S	$\langle 0, 3, 2 \rangle$	$\langle 3 \rangle$	
$\langle R, F, F \rangle$	T	$\langle 0, 3 \rangle$	$\langle 3 \rangle$	✓





$$\forall\, \psi_t \in \Psi_{\{CCW,CW\}} \Rightarrow s_t = (\psi_t + s_0^A) \ \mathbf{mod}\ 4$$

$$S_{l+1} \leftarrow \langle s_t \rangle$$

$$Z_l \leftarrow \langle MrCC, s_0^A \rangle$$

Construction of coarser F4 chain code

Algorithm 1 Construct coarser chain code

procedure ConstructCoarser(S_l)
    $S_l' \leftarrow$ AdjustStartingPoint(S_l)
    $S_l'' \leftarrow$ RemoveDanglingEdges(S_l')
    $S_{l+1}, Z_l, A \leftarrow \emptyset$
    while $|S_l''| > 0$ **do**
        $Q^A \leftarrow$ GetQuadruple(S_l'', A)
        $\langle s_0^A, \delta_1, \delta_2, \delta_3 \rangle \leftarrow \Delta(Q^A)$ ▷ use (5)
        $z, \Psi, A \leftarrow$ LookUp($\langle \delta_1, \delta_2, \delta_3 \rangle$)
        $Z_l \leftarrow Z_l \cup \langle z \rangle$ ▷ append z to Z_l
        for all $\psi_t \in \Psi$ **do**
            $s_t \leftarrow (\psi_t + s_0^A) \bmod 4$ ▷ use (6)
            $S_{l+1} \leftarrow S_{l+1} \cup \langle s_t \rangle$ ▷ append s_t to S_{l+1}
        end for
    end while
    return Z_l, S_{l+1}
end procedure

Progressive decoding

Algorithm 2 Decode MrCC layer

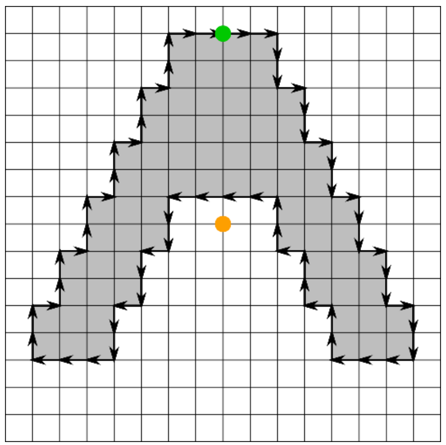
procedure ConstructF4(Z_l, S_{l+1})
  $Adjust_{prev} \leftarrow$ **false**
  $S_l \leftarrow \emptyset$
  for all $z_k \in Z_l$ **do**
    $\langle \delta_1, \delta_2, \delta_3 \rangle, |\Psi|, IsAdjust \leftarrow \text{LookUp}(z_k)$
    if not(IsProvided(s_0^A)) **then**
      $S^t \leftarrow \text{GetNextSubsequence}(S_{l+1}, |\Psi|)$
      $s_0^A = (s_0^t - \psi_0 + 4) \bmod 4$ ▷ use (7)
    end if
    $Q^A \leftarrow \Delta^{-1}(\langle s_0^A, \delta_1, \delta_2, \delta_3 \rangle)$ ▷ invert (5)
    $Q^A \leftarrow \text{ClearAdjustment}(Q^A, Adjust_{prev})$
    $S_l \leftarrow S_l \cup Q^A$ ▷ append Q^A to S_l
    $Adjust_{prev} \leftarrow IsAdjust$
  end for
  return S_l
end procedure

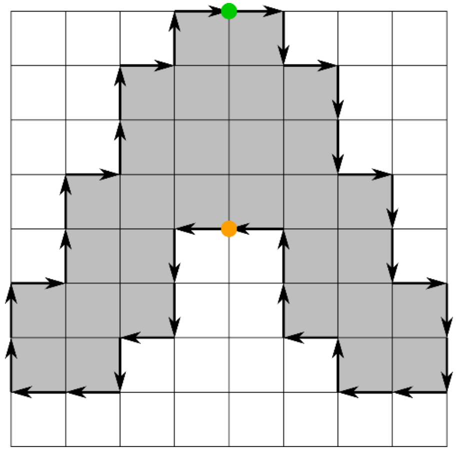
Example

$$S_0 = \langle 3,0,0,3,3,0,3,3,0,3,3,0,3,3,0,3,3,0,3,3, 2,2,2,1,1,2,1,1,2,1,1, 2,2,2,2,3,3,2,3,3,2,3,3, 2,2,2,1,1,0,1,1,0,1,1,0,1,1,0,1,1,0,1,1,0,0,1 \rangle$$

$$S_1 = \langle 0,3,0,3,3,0,3,3,0,3,3,2,2,1,2,1,1,2,2,3, 3,2,3,2,2,1,1,0,1,1,0,1,1,0,1,0 \rangle$$

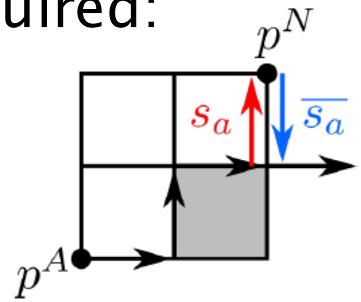
$$Z_0 = \langle E,3,B,3,H,2,B,3,H,2,B,3,B,2, E,1,B,1,H,0,F,1,G,1,L,3,C,2,T,3,B,1, C,0,L,1,C,0,L,1,C,0,R,1,l \rangle$$





Lookup table – appendix

- $\delta_1 = F \Rightarrow \Psi_{CW} = \langle 0 \rangle$;
- $\delta_1 = L \Rightarrow \Psi_{CW} = \langle 1,0,3 \rangle_{|0..3}$;
- $\delta_1 = R \Rightarrow \Psi_{CW} = \langle 0,3,2 \rangle_{|0..3}$;
- $\delta_3 = F \Rightarrow$ remainder $A = \langle \overline{s_a}, s_3^A \rangle$ is required:
  - $\Delta(Q^{A+}) = \begin{cases} \langle s_0^A, \delta_1, \delta_2, L \rangle; & CW \\ \langle s_0^A, \delta_1, \delta_2, R \rangle; & CCW \end{cases}$



- Based on above:
  - $\psi_0^{CW} = \begin{cases} 0; & \delta_1 = F \vee \delta_1 = R \\ 1; & \delta_1 = L \end{cases}$
  - $\psi_i^{CW} = \left(\psi_{i-1}^{CW} + 3\right) \mathbf{mod}\, 4$
- Of course, we must determine $|\Psi|(\Delta)$;

$\langle \delta_1, \delta_2, \delta_3 \rangle$	Ψ_{CW}	$A = \langle \overline{s_a}, s_3^A \rangle$
$\langle F, \delta_2, \delta_3 \rangle$	$\langle 0 \rangle$	
$\langle L, R, F \rangle$	$\langle 1, 0 \rangle$	✓
$\langle L, R, L \rangle$	$\langle 1, 0 \rangle$	
$\langle L, R, R \rangle$	$\langle 1, 0, 3 \rangle$	
$\langle L, F, R \rangle$	$\langle 1, 0 \rangle$	
$\langle L, F, F \rangle$	$\langle 1 \rangle$	✓
$\langle L, F, L \rangle$	$\langle 1 \rangle$	
$\langle L, L, R \rangle$	$\langle 1 \rangle$	
$\langle L, L, F \rangle$	$\langle \rangle$	✓
$\langle L, L, L \rangle$	$\langle \rangle$	
$\langle R, L, F \rangle$	$\langle 0 \rangle$	✓
$\langle R, L, L \rangle$	$\langle 0 \rangle$	
$\langle R, L, R \rangle$	$\langle 0, 3 \rangle$	
$\langle R, R, R \rangle$	$\langle \rangle$	
$\langle R, R, F \rangle$	$\langle 0, 3, 2 \rangle$	✓
$\langle R, R, L \rangle$	$\langle 0, 3, 2 \rangle$	
$\langle R, F, L \rangle$	$\langle 0, 3 \rangle$	
$\langle R, F, R \rangle$	$\langle 0, 3, 2 \rangle$	
$\langle R, F, F \rangle$	$\langle 0, 3 \rangle$	✓

Lookup table

- Map subsequence of $\Delta \mapsto \mathbb{Z}$:
  - $\lambda(\delta) = \begin{cases} -1; & \delta = R \\ 0; & \delta = F \\ 1; & \delta = L \end{cases}$
  - $\tau(\Delta) = \begin{cases} 0; & \lambda(\delta_1) = 0 \\ \alpha\lambda(\delta_1) + \beta\lambda(\delta_2) + \gamma\lambda(\delta_3); & \text{otherwise} \end{cases}$
- α must be large enough for $\delta_1 = R \Rightarrow \tau < 0$ and $\delta_1 = L \Rightarrow \tau > 0$
- Observing possible options ($\beta = \gamma; \beta = \nu, \gamma = \nu + \xi; \beta = \nu + \xi, \gamma = \nu$):
  1. $\alpha = \frac{7}{2}, \beta = 2, \gamma = \frac{1}{2}$
  2. $\alpha = \frac{7}{2}, \beta = 1, \gamma = \frac{3}{2}$

$\langle \delta_1, \delta_2, \delta_3 \rangle$	τ_1	τ_2
$\langle F, \delta_2, \delta_3 \rangle$	0	0
$\langle L, L, L \rangle$	6	6
$\langle L, R, R \rangle$	1	1
$\langle L, R, L \rangle$	2	4
$\langle L, F, R \rangle$	3	2
$\langle L, F, L \rangle$	4	5
$\langle L, L, R \rangle$	5	3
$\langle R, R, R \rangle$	-6	-6
$\langle R, R, L \rangle$	-5	-3
$\langle R, F, R \rangle$	-4	-5
$\langle R, L, R \rangle$	-2	-4
$\langle R, F, L \rangle$	-3	-2
$\langle R, L, L \rangle$	-1	-1

Lookup table

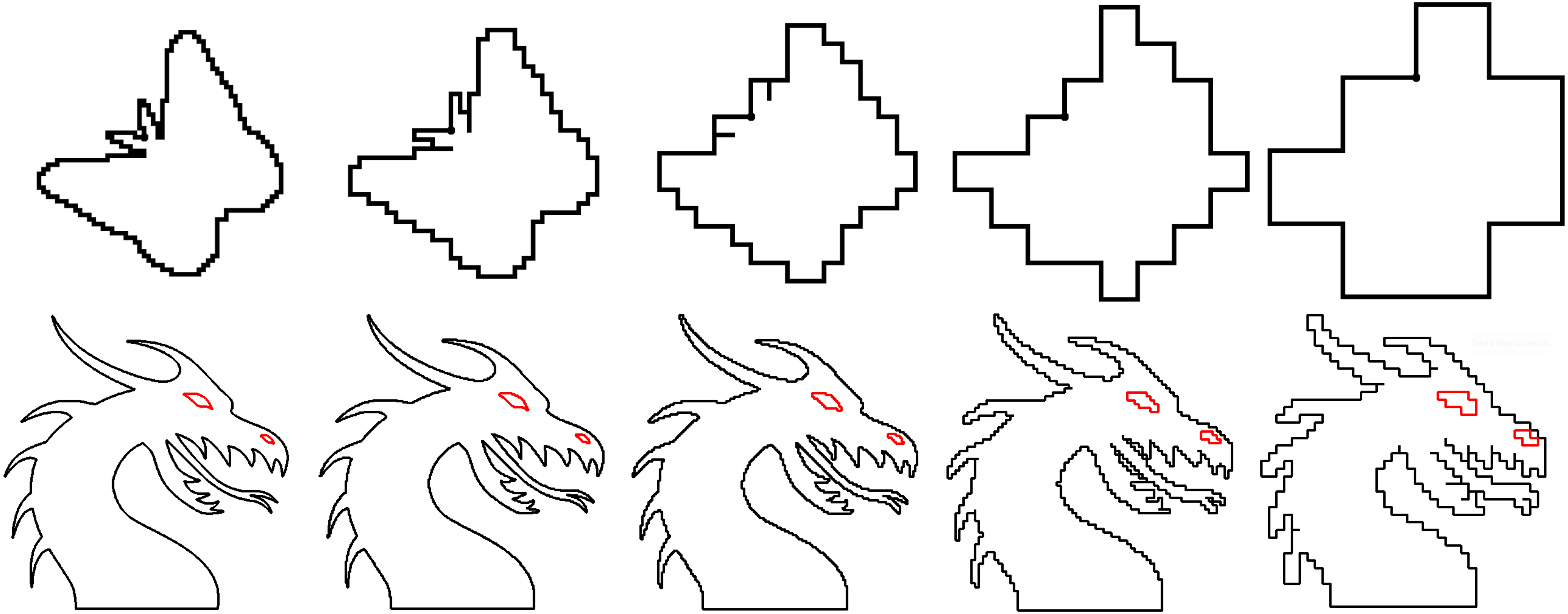
- $|\Psi_{CW}|(\Delta) = \rho(\Delta) = \begin{cases} \left(\left\lfloor \frac{-\tau_1(\Delta)}{2} \right\rfloor + 1\right) \mathbf{mod}\, 4; & \tau_1(\Delta) \leq 0 \\ 3 - \lfloor \frac{\tau_1(\Delta)}{2} \rfloor; & x > 0 \end{cases}$
- $|\Psi_{CW}|(\Delta) = \rho(\Delta) = \begin{cases} 1; & \tau_2(\Delta) = 0 \\ \left(-\tau_2(\Delta) + 2\left\lfloor \frac{-\tau_2(\Delta)}{4} \right\rfloor\right) \mathbf{mod}\, 4; & \tau_2(\Delta) < 0 \\ 4 - \tau_2(\Delta) + 2\left\lfloor \frac{\tau_2(\Delta)}{4} \right\rfloor; & \tau_2(\Delta) > 0 \end{cases}$
- $\psi_0^{CW} = \begin{cases} 0; & \tau \leq 0 \\ 1; & \tau > 0 \end{cases}$
- $\psi_i^{CW} = \left(\psi_{i-1}^{CW} + 3\right) \mathbf{mod}\, 4$

$\tau(\Delta)$	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$\lvert\Psi_{CW}\rvert$	0	3	3	2	2	1	1	3	2	2	1	1	0
$\lvert\Psi_{CW}\rvert$	0	3	2	3	2	1	1	3	2	1	2	1	0

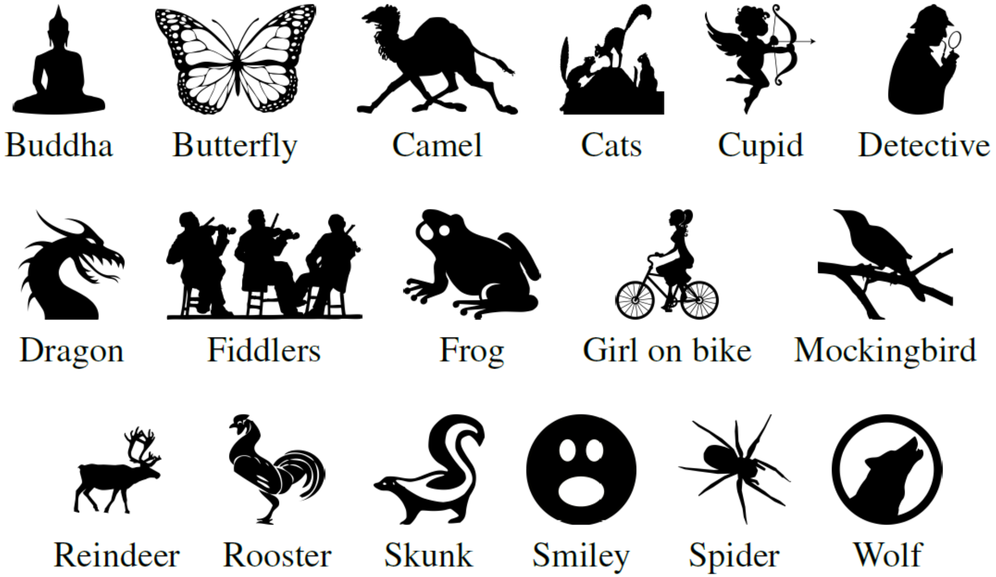
Huffman codes

MrCC	Probability [%]	Huffman code
A	28.917	10
B	5.613	0110
C	10.021	000
D	0.034	11010011001
E	4.536	11011
F	8.277	1111
G	1.556	011101
H	2.871	01111
I	0.167	1101001101
J	0.465	11010010
K	0.381	11010000
L	6.195	1100
M	0.248	110100111
N	10.146	010
O	0.001	110100110000
P	0.019	110100110001
Q	2.565	00101
R	5.190	0011
S	1.137	001000
T	8.192	1110
i	1.770	110101
j	0.047	001001
k	1.264	011100
l	0.388	11010001

Results



Results



Shape	$\lvert S_0 \rvert$	Holes	L	No. of F4 bits	No. of MrCC bits	Ratio[%]
Buddha	11366	1	11	45484	37200	82
Butterfly	64448	126	12	266416	226645	85
Camel	20156	9	12	80996	63261	78
Cats	22238	5	12	89052	68502	77
Cupid	25160	4	11	100636	74565	74
Detective	14128	2	11	56636	43846	77
Dragon	26334	2	12	105112	78040	74
Fiddlers	27050	25	12	109756	87880	80
Frog	20646	4	12	82364	64896	79
Girl on bike	50752	52	12	206656	170067	82
Mockingbird	17800	4	12	71396	55208	77
Reindeer	19864	4	12	79284	62228	78
Rooster	32894	9	11	131356	102941	78
Skunk	23960	3	12	96088	74376	77
Smiley	16210	3	12	65096	50088	77
Spider	23900	0	12	95116	73512	77
Wolf	22058	1	12	88264	66765	76

Bibliography

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  Andrej Nerat, Damjan Strnad, Krista Rizman Žalik, Borut Žalik. **An effcient multi-resolution chain coding**. IEEE Access, 2024, 11 pp.

- Image analysis & stereology (Q4; while sending Q2 ☹):

  Andrej Nerat, Damjan Strnad, Eva Zupančič, Borut Žalik. **Extended algorithm to construct a quadtree from Freeman chain code in four directions.** Image analysis & stereology : official journal of the International Society for Stereology. 2019, 227-235 (Scopus: 2 citations, 1 citation without self-citations).