FACULTY OF APPLIED SCIENCES
UNIVERSITY
OF WEST BOHEMIA

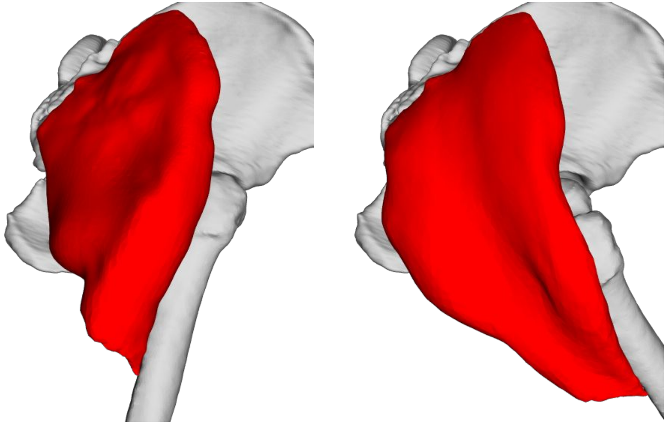
STUDENT SCIENTIFIC CONFERENCE 2023

# A generic model of hyperspace curvature preservation for a dynamic radial basis function implicit surface

Ing. Martin Červenka,
Faculty of Applied Sciences,
Department of Computer Science and Engineering,
25th of May 2023

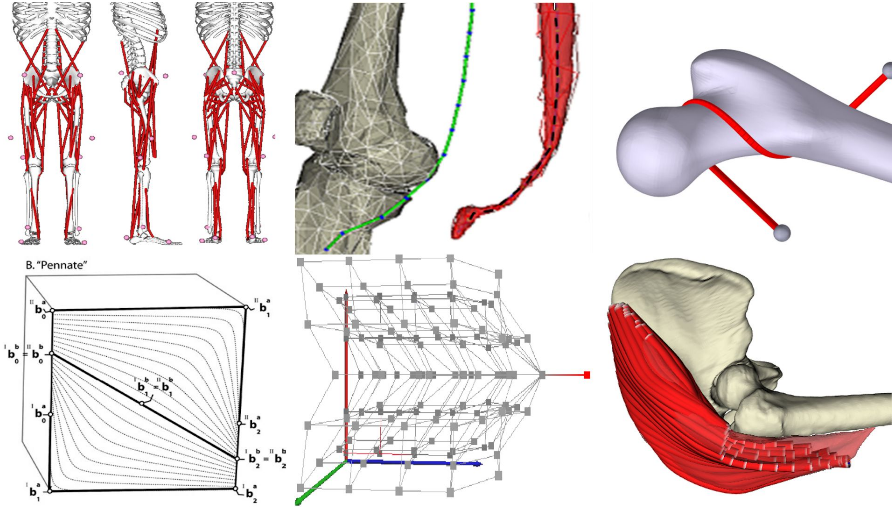
# Introduction to the problem

- Computerised muscle modelling
  - What is known (all of that are rough estimates):
    - initial position and shape of the triangular muscle mesh
    - muscle-bone relations, such as surface areas, where the muscle is attached to a bone
    - bone movements
  - The problem: estimating the muscle location and shape during the bone movement
  - => Inverse kinematics



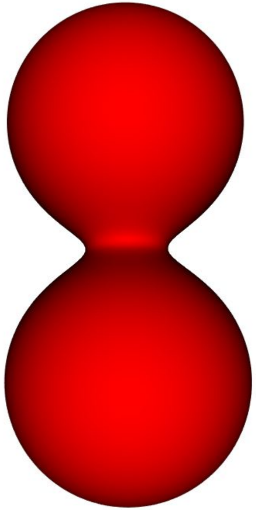
# Our contributions to the muscle modelling

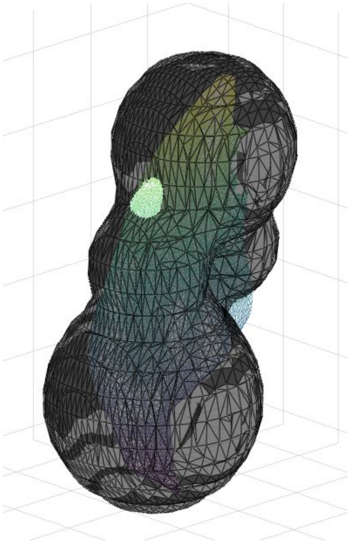
- Our methods build upon triangular meshes so far
- Previous work:
    - Via-points[1], Wrapping obstacles[2], Mass-spring system[3]
- Our contributions:
    - PBD (position-based dynamics) surface modeling approach [4]
        - with various collision detection algorithms [4][5]
    - Also working concurrently on ARAP (As-Rigid-As-Possible) surface modelling
    - possible combination of ARAP & PBD?
- **Why just triangular meshes though?**



# Radial basis functions

- The muscle may be represented differently
  - implicit surface approximation
- Radial basis functions - RBFs
  - weighted sum of individual RBFs
  - weights can be calculated
  - produces smooth approximation
  - if well selected (Gaussian), then infinitely differentiable

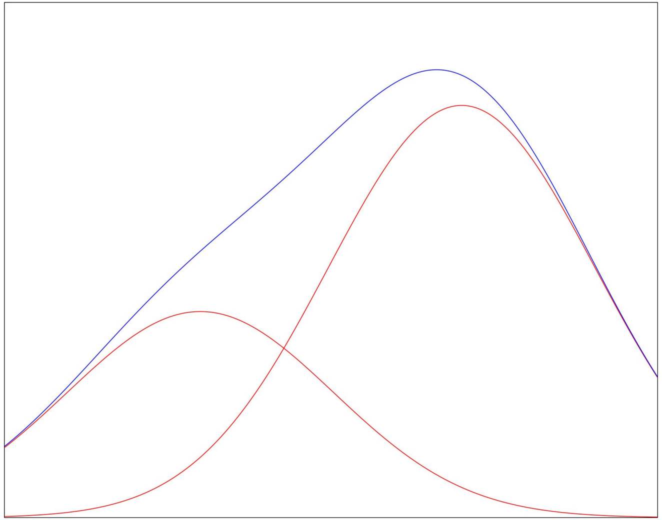




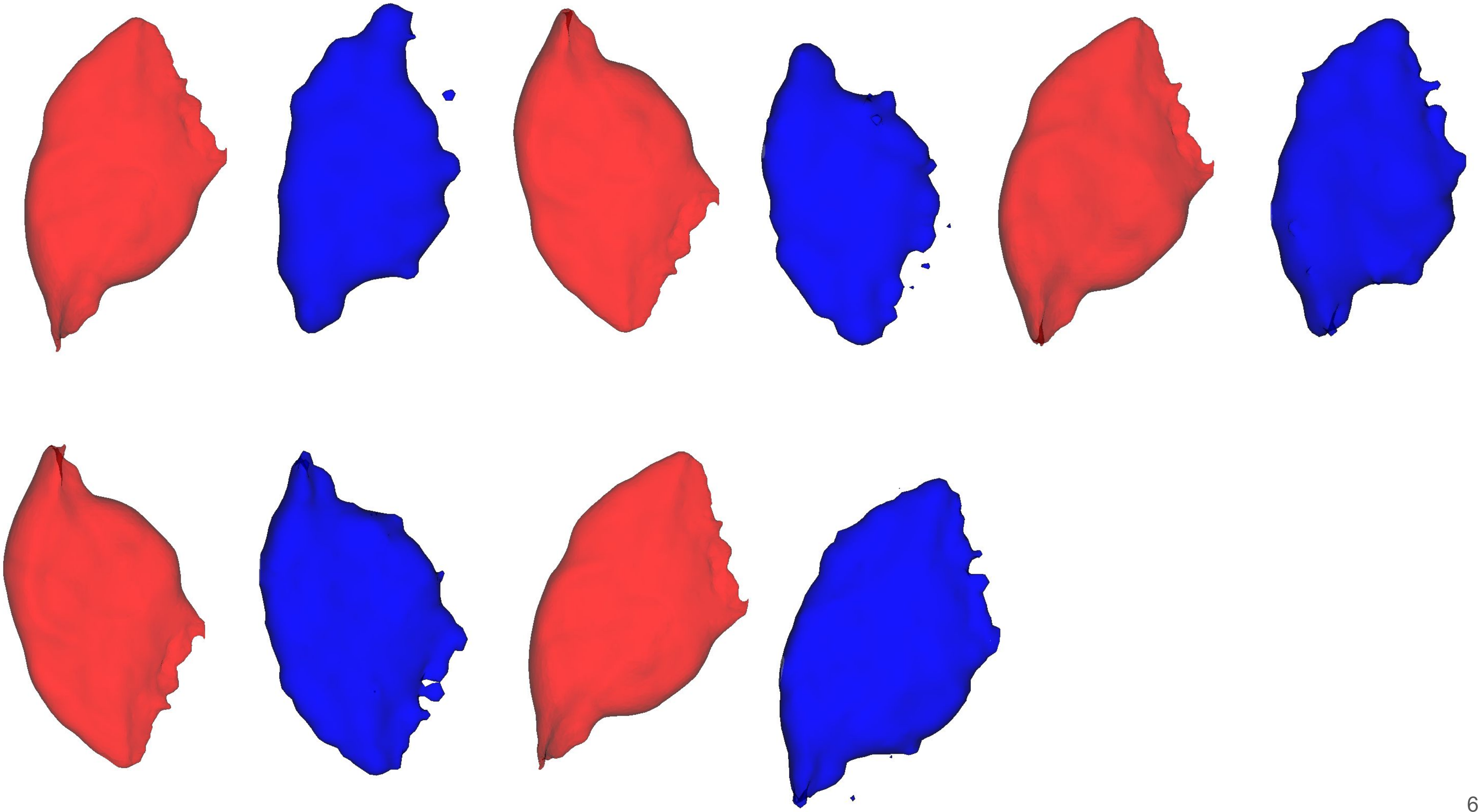
# Gaussian RBF

$\alpha$ - shape parameter (global)
$\lambda_i$ - weight of the individual RBF
$\xi_i$ - centre of the individual RBF

$$f(\mathbf{x}) = \sum_{i=1}^{N} \lambda_i e^{-\alpha||\mathbf{x}-\xi_i||_2^2}$$



- find suitable
  - number of RBFs (depends on desired precision)
  - shape parameter (see e.g. [6])
  - centre points (goal of this work)



# Finding the new centre locations

- Define the cost function (difference between the original and new curvature over the whole interval)
- Obtain the gradients with respect to all ξ.
- Put everything together

$$C_f = \int_{\mathbb{R}^d} ||\varkappa_f - \varkappa_{f_{\text{init}}}||_2^2 d\mathbf{x}$$

$$\nabla \varkappa_f = \left[ \frac{\partial \varkappa}{\partial \xi_{i1}} \quad \frac{\partial \varkappa}{\partial \xi_{i2}} \quad \frac{\partial \varkappa}{\partial \xi_{i3}} \quad \dots \right]$$

$$\nabla C_{fj} = \frac{8\alpha^2}{d} \int_{\mathbb{R}^d} (\varkappa_f - \varkappa_{f_{\text{init}}}) \sum_{i=1}^{N} g_i(\mathbf{x}) (x_j - \xi_{ij}) (2\alpha ||\mathbf{x} - \xi_{\mathbf{i}}||_2^2 - 2 - d) d\mathbf{x}$$

# Mean curvature preservation

- Curvature in hyperspace (3D in this case)
- Defined as the mean eigenvalue of the Hessian

$$\varkappa(\mathbf{H}) = \frac{1}{d}\sum_{i=1}^{d}\lambda_i$$

$$\mathbf{H}(f(\mathbf{x})) = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} & \cdots \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} & \cdots \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}$$

# Conclusion & Future work

- The theoretical model of finding the centre points is as follows:

$$\nabla C_{fj} = \frac{8\alpha^2}{d} \int_{\mathbb{R}^d} \left(\varkappa_f - \varkappa_{f_{\text{init}}}\right) \sum_{i=1}^{N} g_i\left(\mathbf{x}\right) \left(x_j - \xi_{ij}\right) \left(2\alpha||\mathbf{x} - \xi_{\mathbf{i}}||_2^2 - 2 - d\right) d\mathbf{x}$$

- The future work is to implement the theoretical model into the muscle modelling framework

# References


- **[1]** Hájková, J., Kohout, J.: Human body model movement support: Automatic muscle control curves computation. pp. 196–211 (05 2014). https://doi.org/10.1007/978-3-319-07148-0_18
- **[2]** Lloyd, J.E., Roewer-Després, F., Stavness, I.: Muscle path wrapping on arbitrary surfaces. IEEE Transactions on Biomedical Engineering 68(2), 628–638 (2021). https://doi.org/10.1109/TBME.2020.3009922
- **[3]** Janák., T., Kohout., J.: Deformable muscle models for motion simulation. In: Proceedings of the 9th International Conference on Computer Graphics Theory and Applications - GRAPP, (VISIGRAPP 2014). pp. 301–311. INSTICC, SciTePress (2014). https://doi.org/10.5220/0004678903010311
- **[4]** Cervenka, M., Kohout, J.: Fast and Realistic Approach to Virtual Muscle Deformation, in Proceedings of the 13th International Joint Conference on Biomedical Engineering Systems and Technologies - Volume 5: HEALTHINF, ISBN 978-989-758-398-8, pages 217-227. (2020), https://doi.org/10.5220/0009129302170227
- **[5]** Havlicek, O., Cervenka, M., Kohout, J.: Collision detection and response approaches for computer muscle modelling, Informatics 2022, IEEE proceedings, pp.120-125, ISBN 979-8-3503-1034-4, Poprad, Slovakia, (2022)
- **[6]** Cervenka, M., Skala, V.: Conditionality Analysis of the Radial Basis Function Matrix, ICCSA 2020 proceedings, part II, LNCS, pp. 30-43, Springer, (2020) https://doi.org/10.1007/978-3-030-58802-1_3

# Thank you for your attention