Priority-based encoding of triangle mesh connectivity for a known geometry
(and beyond)

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## Connectivity compression with known geometry

Geometry encoded first

- Temporal prediction
- Predictable spatial structure
- Multiple-rate compression



## Conventional connectivity coding

Simple, does not require geometry information


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## Causes reordering!

Permutation map: $\frac{1}{F} \log _{2}(V!)$ bpf

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- Symbol $\mathbf{0}$ reserved for boundary



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## Priority-driven traversal

Triangles where encoder most certain processed first

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- More feasible situation in future
- Exponential distribution expected
- exp-Golomb code + CABAC
- Benefits when smaller values are encoded first



## Fixed vs. Priority-driven traversal



Distance-ranked


Priority-driven

Fixed vs. Priority-driven traversal


Priority-driven

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\begin{gathered}
q=\theta-w_{1} \cdot \bar{d}+w_{2} \cdot \phi+w_{3} \cdot S \\
w_{1}, w_{2}, w_{3} \in \mathbb{R}_{>0}
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- Optimally - regular planar triangulation


## Penalize long triangles



Best value: $\pi$

## Distance from parallelogram prediction

Planarity, similar properties (area, edge length, inner angles)


$$
\bar{d}=\frac{d}{l_{\mathrm{avg}}}
$$

Best value: 0

## Planarity



$$
\phi=\pi-\arccos \left(\mathbf{n}_{b} \cdot \mathbf{n}_{c}\right)
$$

Best value: $\pi$

## Triangle similarity

Alternative to parallelogram rule without enforced planarity


$$
\begin{aligned}
r_{s} & =s_{b} / s_{v}, \quad r_{l}=l_{b} / l_{c} \\
r & =\left(r_{s}+r_{l}\right) / 2 \\
S & =-\left(\left|r-r_{s}\right|+\left|r-r_{l}\right|\right) / 2
\end{aligned}
$$

Best value: 0

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- $q$ does not behave like distances

Still can be evaluated by radial search, the search area can be deduced by plugging optimal values into the equation for evaluating $q$.

$$
q=\theta-w_{1} \cdot \bar{d}+w_{2} \cdot \phi+w_{3} \cdot S
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$$
q_{c}=\theta_{\min }-w_{1} \cdot 0+w_{2} \cdot \pi+w_{3} \cdot 0
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q=\theta-w_{1} \cdot \bar{d}+w_{2} \cdot \phi+w_{3} \cdot S
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$$
q_{c}=\pi-w_{1} \cdot \bar{d}_{\max }+w_{2} \cdot \pi+w_{3} \cdot 0
$$

$$
d_{\max }=\frac{\left(w_{2} \cdot \pi+\pi-q_{c}\right) \cdot l_{\mathrm{avg}}}{w_{1}}
$$



All vertices with $q \geq q_{c}$ lie within $\mathcal{B}_{d} \cap \mathcal{B}_{\theta}$

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- Evaluate $q_{\text {max }}$ of each edge
- Find $q_{t}$ which best separates $q_{\text {max }}$ of inner/boundary edges
- Predict boundary if $q_{\max }<q_{t}$

| Prediction | Actual | Symbol |
| :--- | :--- | :---: |
| Inner | Inner | $i$ |
| Boundary | Boundary | 0 |
| Boundary | Inner | $i+1$ |
| Inner | Boundary | $\max (i)+1$ |

## Experimental results

## Main experiment



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Proposed $\quad 1.112 \mathrm{bpf}$

Distance-ranked 1.343 bpf $H=1.317$

$\begin{array}{ll}0.152 \mathrm{bpf} & 1.050 \mathrm{bpf} \\ H=0.171 & H=1.195\end{array}$
$0.264 \mathrm{bpf} \quad 2.261 \mathrm{bpf}$
$H=0.232$


$$
\begin{aligned}
& 0.181 \mathrm{bpf} \\
& H=0.282
\end{aligned}
$$

$$
\begin{aligned}
& 0.331 \mathrm{bpf} \\
& H=0.381
\end{aligned}
$$



$$
\begin{aligned}
& 0.448 \mathrm{bpf} \\
& H=0.521
\end{aligned}
$$

$$
\begin{aligned}
& 0.708 \mathrm{bpf} \\
& H=0.693
\end{aligned}
$$

0.919 bpf
$H=1.079$

1.523 bpf
$H=1.552$

The Distance-ranked algorithm does not achieve a consistently lower data rate than $H$.

## Main experiment



## Mesh compression

Combined with PC codec [Merry et al. 2006] vs. Weighted parallelogram [Váša-Brunnett 2013]



## Limitations \& Future Work

## PAC-MAN configuration ©...






## Optimal parameters

Difficult to find optimal $w_{1}, w_{2}, w_{3}$ for a certain model.

■ Global trend towards a region of satisfactory rates

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b p f\left(w_{1}, w_{2}, w_{3}\right)
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- Still no guarantee of finding the global minimum


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■ Exhaustive search

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- Still no guarantee of finding the global minimum

In experiments:
■ Default parameters vs. Fine-tuned for each dataset

- Estimated on a subset
- Default parameters still better than Distance-ranked
- No significant improvement for irregular data


## Connection to mesh properties

- Obtain fine-tuned weights for various models


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- Vertex degrees
- Inner angles
- Edge lengths


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So complex, there might not be any.

## Local-frame-based optimization

- Local frame optimization
- Maximize tip vertex quality
- Minimize quality of all other vertices


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Does not consider all the aspects (e.g., priority).

## Quality function for CAD models



## Model-based compression of Time-varying meshes


https://gitlab.kiv.zcu.cz/jdvorak/priority-based-connectivity-coding


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