Vector Field Compression Using Radial Basis Functions

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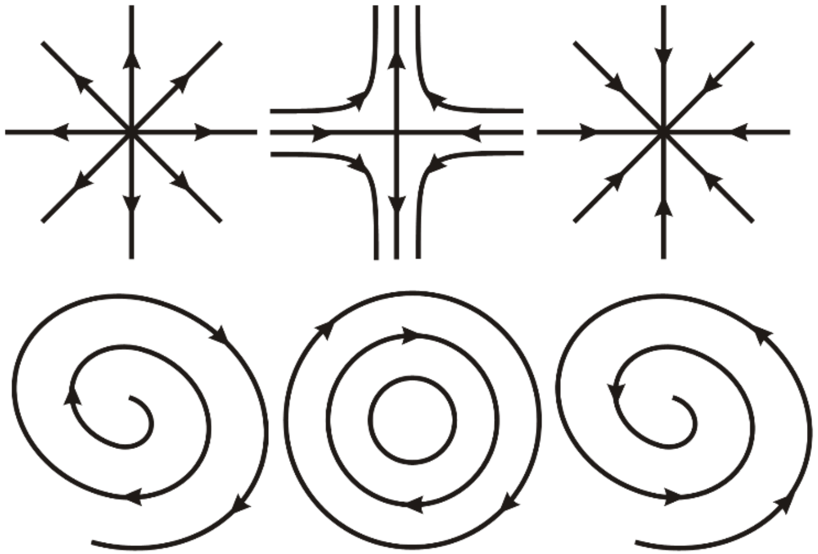
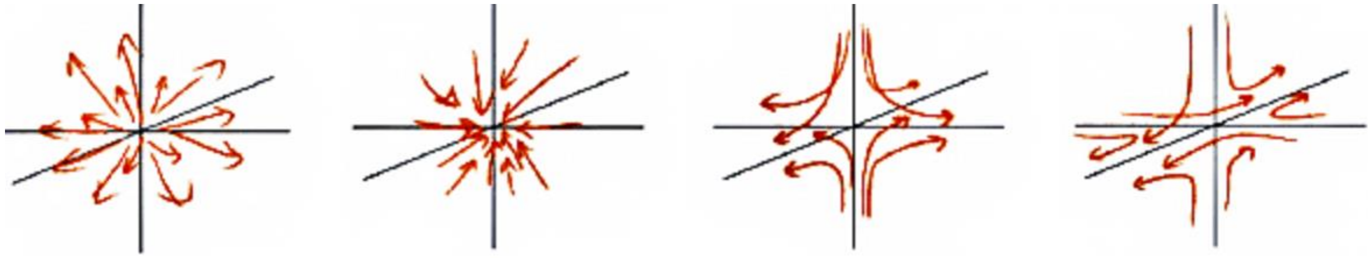
Vector Fields

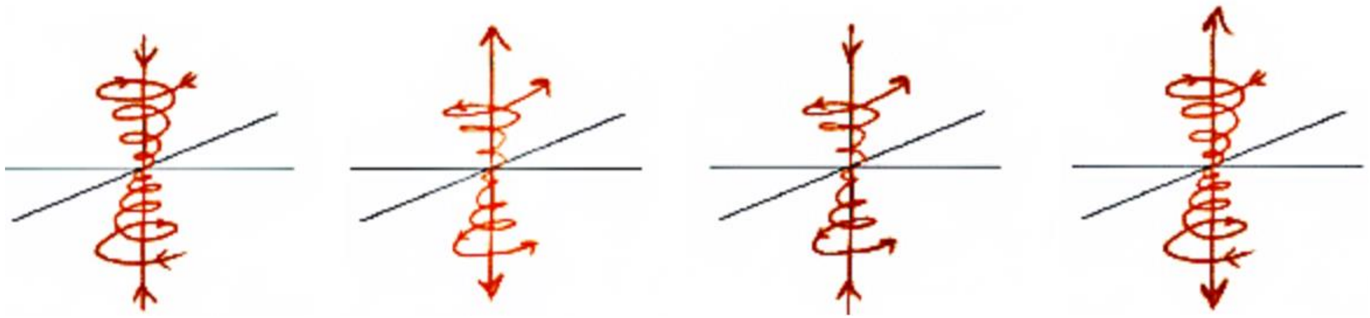
- Vector fields come from numerical simulations or experimental measurements
  - Represented by large data sets
- Vector field is described as
  - $\boldsymbol{v}(\boldsymbol{x},t) = \left[v_x(\boldsymbol{x},t), v_y(\boldsymbol{x},t)\right]^T$ or $\boldsymbol{v}(\boldsymbol{x},t) = \left[v_x(\boldsymbol{x},t), v_y(\boldsymbol{x},t), v_z(\boldsymbol{x},t)\right]^T$
- Critical point ($\boldsymbol{x}_0$)
  - $\frac{dx}{dt} = \boldsymbol{v}(\boldsymbol{x}) = \boldsymbol{0}$, i.e. all components are zero
  - $\frac{dx_\varepsilon}{dt} \neq \boldsymbol{0},\ \forall \boldsymbol{x}_{\boldsymbol{\varepsilon}}: \|\boldsymbol{x}_{\boldsymbol{\varepsilon}} - \boldsymbol{x}_0\| < \boldsymbol{\varepsilon}$
- Linearization of vector field around a critical point (Taylor expansion)
  - $\boldsymbol{v}(\boldsymbol{x}) \approx \boldsymbol{v}(\boldsymbol{x}_0) + \frac{\partial \boldsymbol{v}}{\partial \boldsymbol{x}}(\boldsymbol{x} - \boldsymbol{x}_0)$ → $\begin{bmatrix} v_x \\ v_y \end{bmatrix} = \begin{bmatrix} \frac{\partial v_x}{\partial x} & \frac{\partial v_x}{\partial y} \\ \frac{\partial v_y}{\partial x} & \frac{\partial v_y}{\partial y} \end{bmatrix} \begin{bmatrix} x - x_0 \\ y - y_0 \end{bmatrix}$ → $\boldsymbol{v} = \boldsymbol{J} \cdot (\boldsymbol{x} - \boldsymbol{x}_0)$
  - $\boldsymbol{J}$ is Jacobian matrix

VECTOR FIELDS

- Jacobian matrix
  - Describes vector field around critical point x_0
  - Eigen vectors represent main directions of the flow
  - Eigen numbers
    - Sign defines the direction (sink / source)
    - Imaginary part defines rotation







RADIAL BASIS FUNCTIONS (RBF)

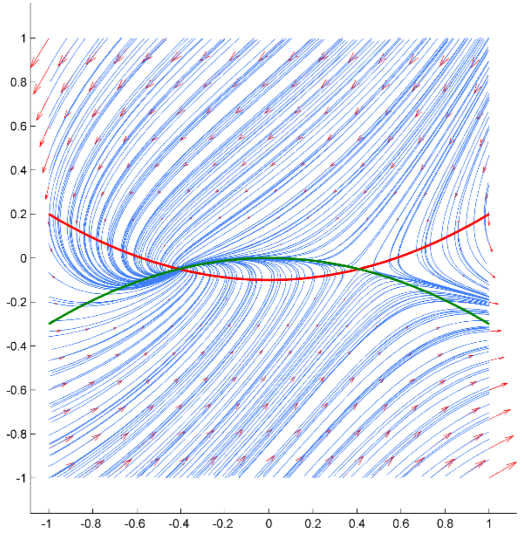
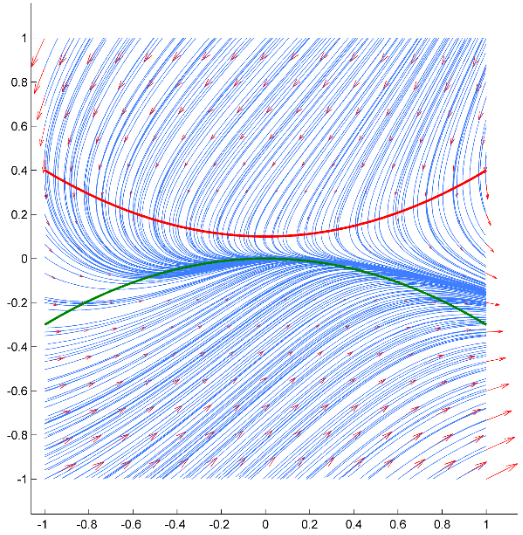
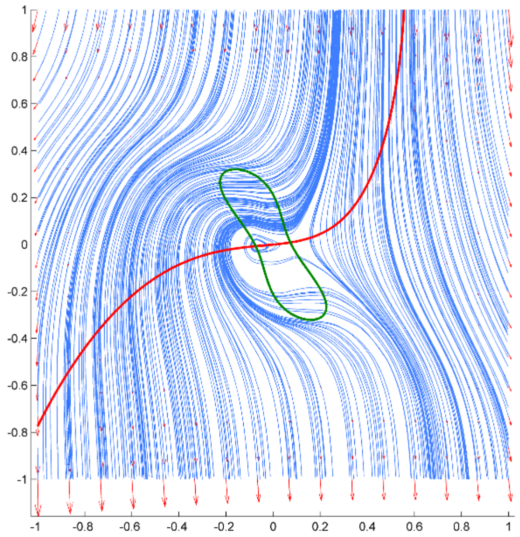
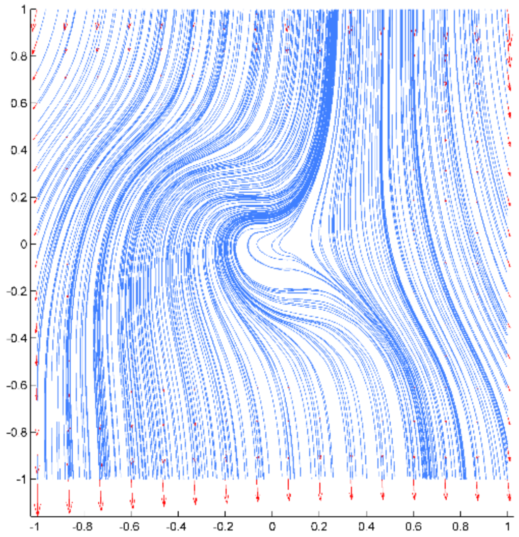
- Mesh-less technique for interpolation/approximation
- Formula of RBF: $f(\boldsymbol{x}) = \sum_{j=1}^{M} \lambda_j \varphi(\|\boldsymbol{x} - \boldsymbol{\xi}_j\|) \quad \rightarrow \quad \boldsymbol{A\lambda} = \boldsymbol{f}$
  - M is number of radial basis functions
  - λ_j are weights
  - $\varphi(r)$ is radial basis function (local / global)
- Approximation of vector field with RBF
  - $\boldsymbol{v}(\boldsymbol{x}) = [v_x(\boldsymbol{x}), v_y(\boldsymbol{x}), v_z(\boldsymbol{x})] = \sum_{j=1}^{M} \boldsymbol{\lambda}_j \varphi(\|\boldsymbol{x} - \boldsymbol{\xi}_j\|) \quad \rightarrow \quad \boldsymbol{A\lambda} = \boldsymbol{v}$
    - $\boldsymbol{v}(\boldsymbol{x})$ is vector
    - $\boldsymbol{\lambda}_j = \left[\lambda_j^{(x)}, \lambda_j^{(y)}, \lambda_j^{(z)}\right]$ is vector

Radial Basis Function

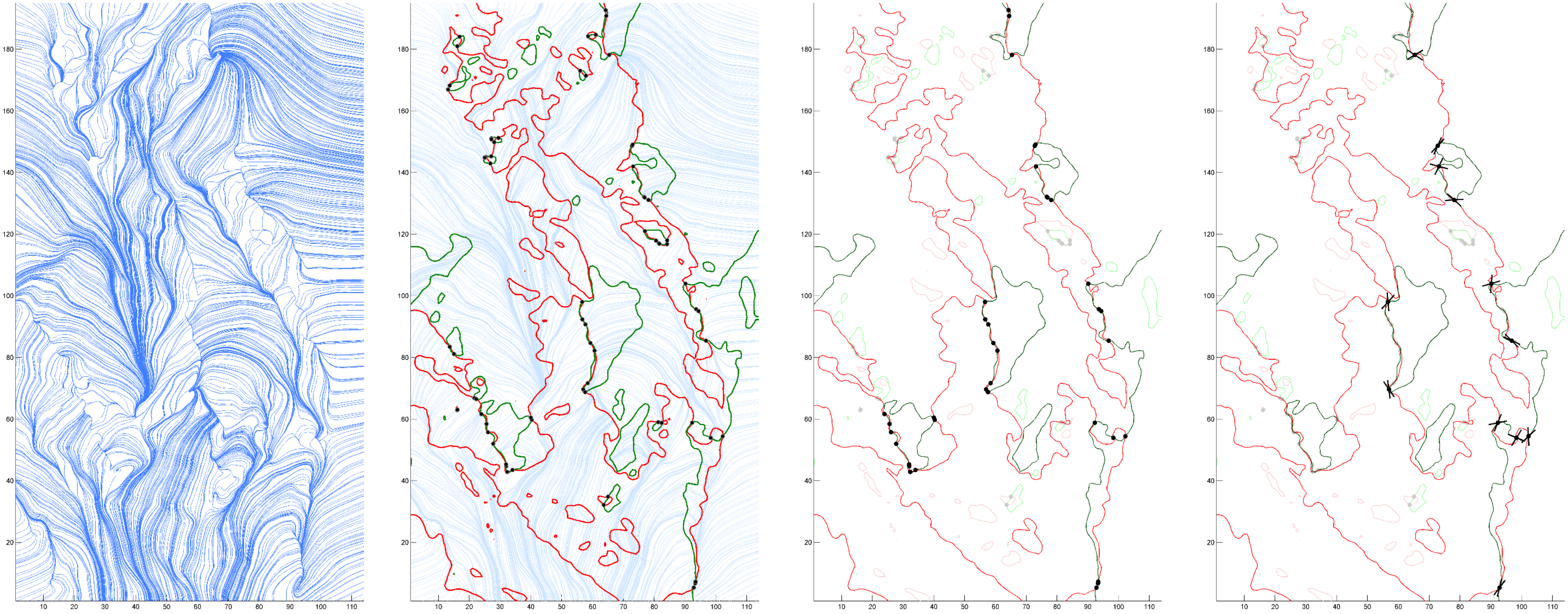
Global RBF		Local RBF
Thin Plate Spline (TPS)	$\varphi(r) = r^2 \log r$	$\varphi_1(r) = (1-\epsilon r)_+$
Gauss function	$\varphi(r) = e^{-(\epsilon r)^2}$	$\varphi_2(r) = (1-\epsilon r)_+^3(3\epsilon r + 1)$
Inverse Quadric (IQ)	$\varphi(r) = \frac{1}{1+(\epsilon r)^2}$	$\varphi_3(r) = (1-\epsilon r)_+^5(8(\epsilon r)^2 + 5\epsilon r + 1)$
Inverse Multiquadric (IMQ)	$\varphi(r) = \frac{1}{\sqrt{1+(\epsilon r)^2}}$	$\varphi_4(r) = (1-\epsilon r)_+^2$
Multiquadric (MQ)	$\varphi(r) = \sqrt{1+(\epsilon r)^2}$	$\varphi_5(r) = (1-\epsilon r)_+^4(4\epsilon r + 1)$
		$\varphi_6(r) = (1-\epsilon r)_+^6(35(\epsilon r)^2 + 18\epsilon r + 3)$
		$\varphi_7(r) = (1-\epsilon r)_+^8(32(\epsilon r)^3 + 25(\epsilon r)^2 + 8\epsilon r + 1)$
		$\varphi_8(r) = (1-\epsilon r)_+^3$
		$\varphi_9(r) = (1-\epsilon r)_+^3(5\epsilon r + 1)$
		$\varphi_{10}(r) = (1-\epsilon r)_+^7(16(\epsilon r)^2 + 7\epsilon r + 1)$

VECTOR FIELD APPROXIMATION – REDUCTION OF CRITICAL POINTS

- For visualization purposes, some critical points are less important than other
  - Important critical points define global character of vector field
  - Less important critical points define only local character of the vector field
- Use of Lagrange multipliers to maintain important critical points

Vector Field Approximation – Reduction of Critical Points



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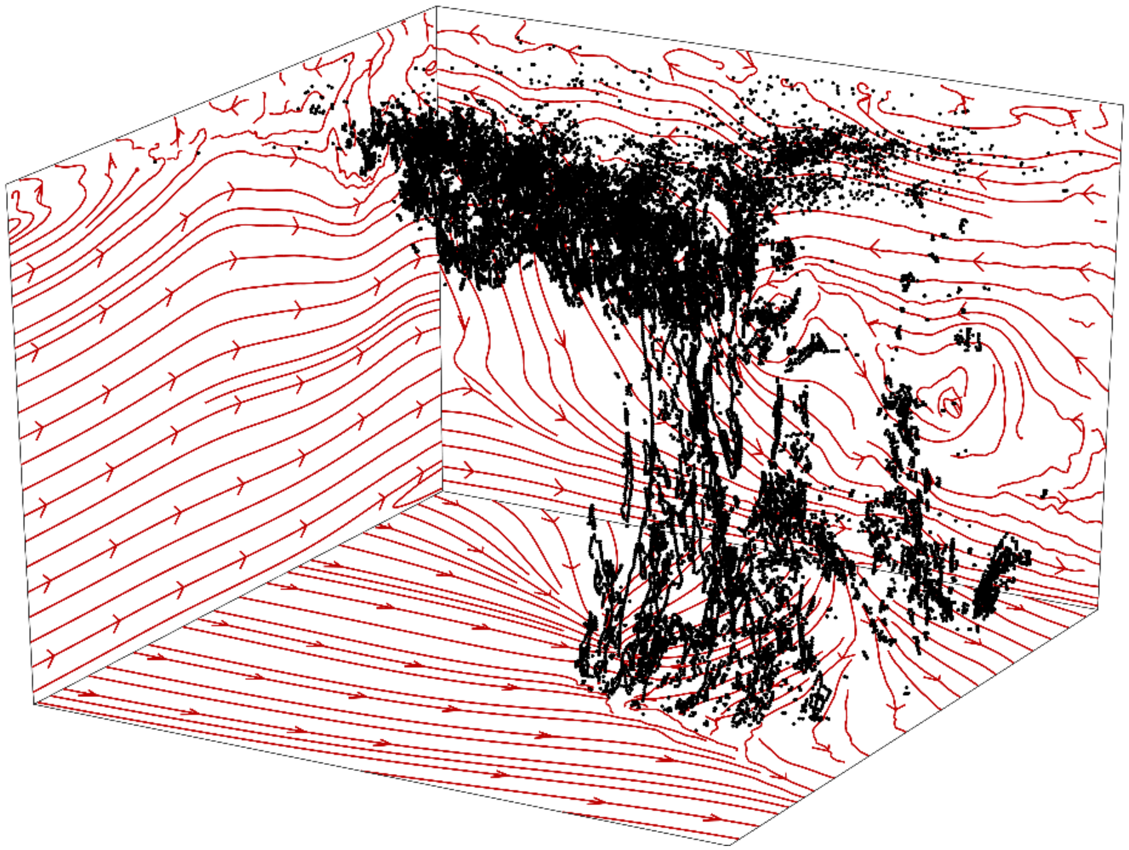
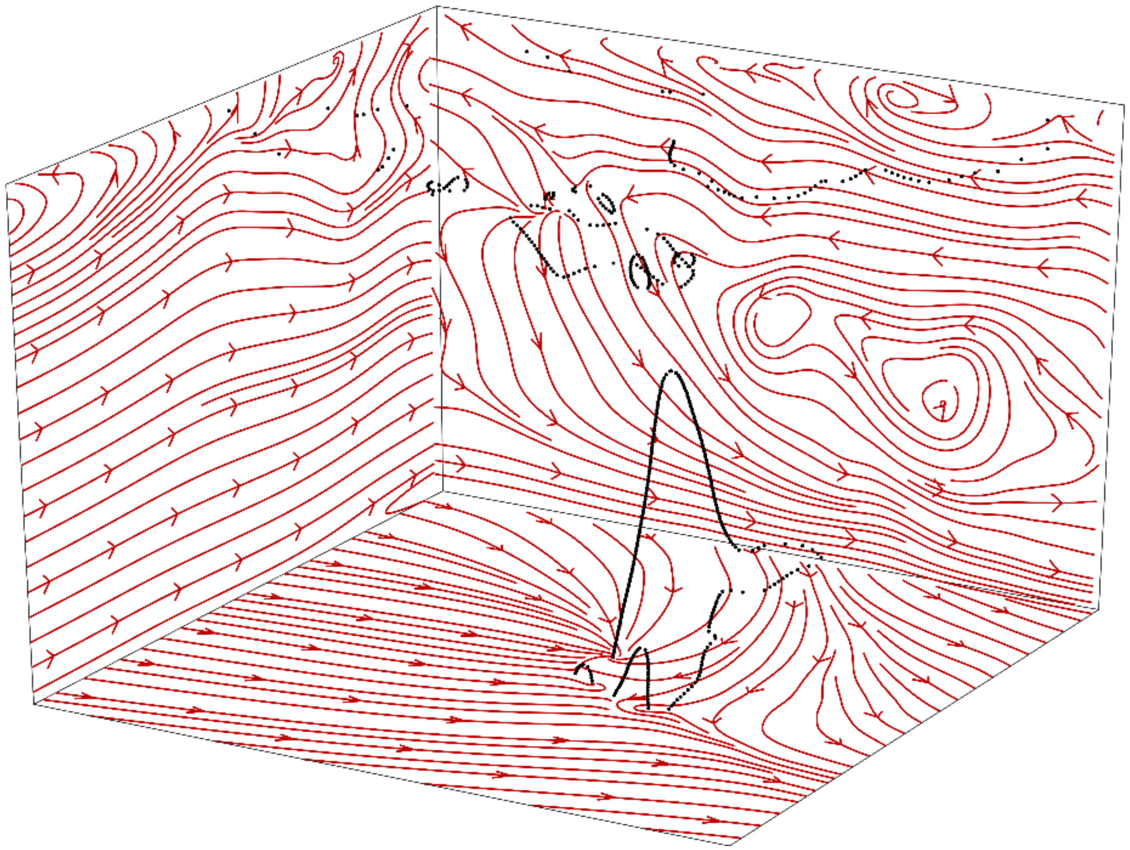


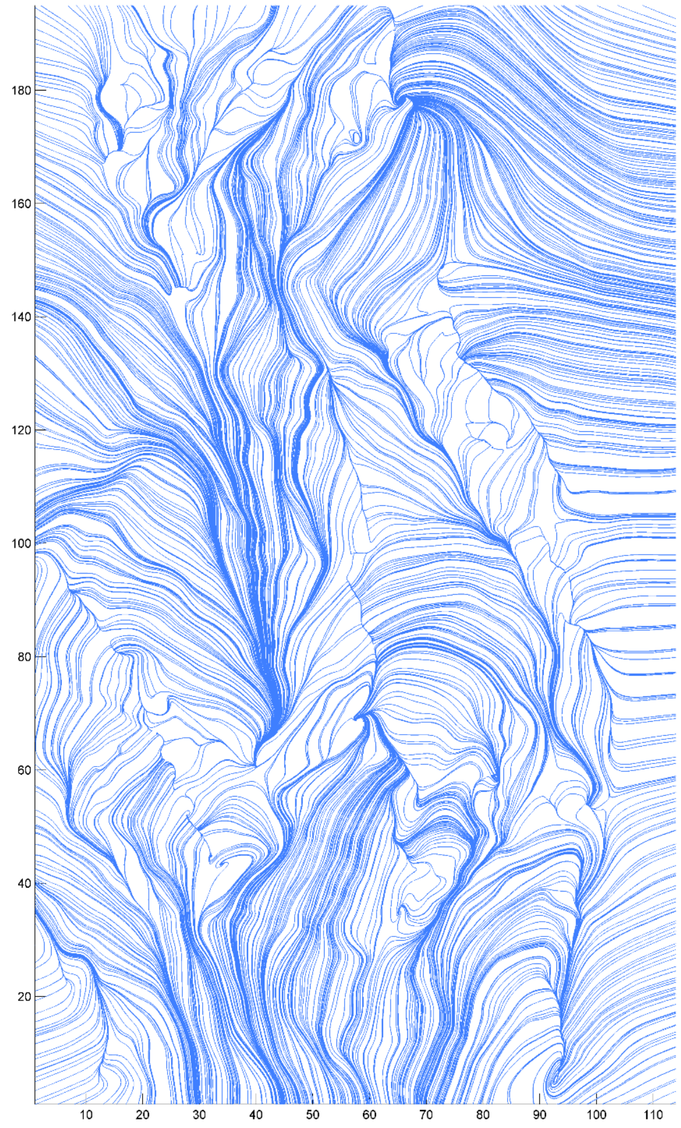


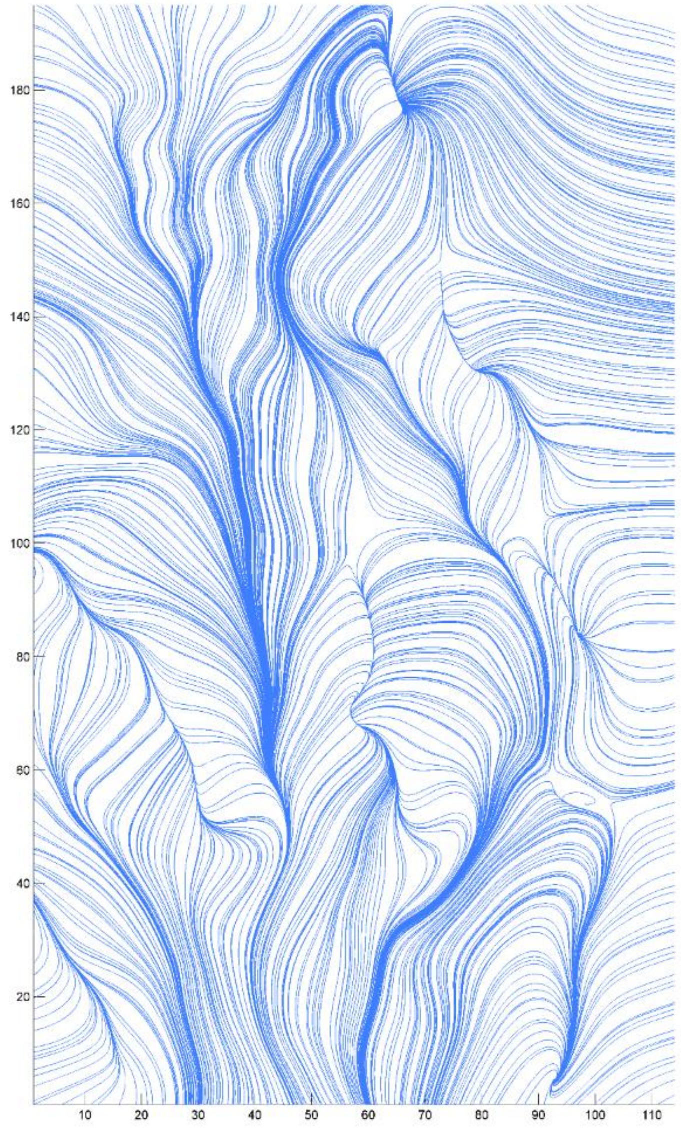
Vector Field Approximation

- Centers of radial basis functions at extremes of v_x and v_y
- Used RBF has elliptical shape
  - Standardly has RBF circular shape
  - It describes local shape of approximated data more precisely
  - Elliptical shape is computed from segmented vector field with Principal component analysis (PCA)

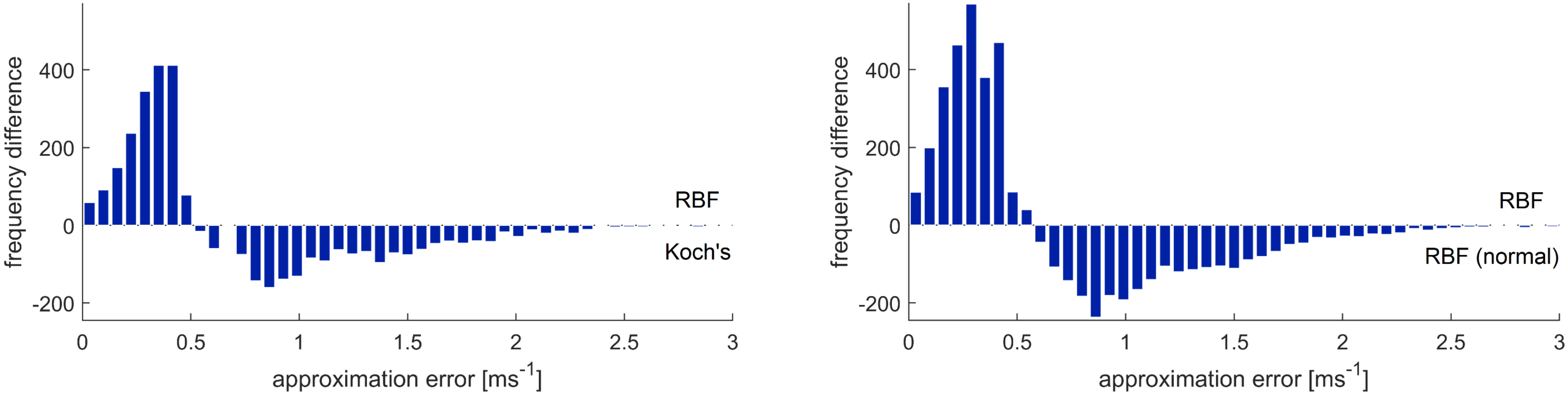
Compression algorithm	Approximation error
Proposed RBF approximation	0.36 ms^{-1}
Koch's algorithm	0.41 ms^{-1}
Standard RBF approximation	0.48 ms^{-1}

Vector Field Approximation





VECTOR FIELD APPROXIMATION



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