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Asymmetric numeral systems (ANS)

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Contents

Introduction

- Similar to arithmetic coding
- One variable: current state (x)
- Also used for redundancy checks
- The sequence is decoded in reversed order

Algorithm procedure

- Set the state to initial value
- For each symbol in input sequence
  - Read the symbol and update the encoder state
  - Renormalize state if needed
- Write the final state to the output stream

Binary ANS coder - encoding

- q – probability of symbol 1
- $x' = C(s,x) = \begin{cases} \left\lceil \frac{x+1}{1-q} \right\rceil, s = 0 \\ \left\lfloor \frac{x}{q} \right\rfloor, s = 1 \end{cases}$

Binary ANS coder - decoding

- $s = \lceil (x+1)q \rceil - \lceil xq \rceil$

- $x' = \begin{cases} x - \lceil xq \rceil, s = 0 \\ \lceil xq \rceil, s = 1 \end{cases}$

Binary ANS coder - example

- Sequence: 1, 1, 0, 1, 0, 0, 0, 1, 0, 1, 1, 1, 1, 0, 1, 0, q = 0.5625
- Starting state: x = 12

1. s = 1: $x' = \left\lfloor \frac{x}{q} \right\rfloor = 21$
2. s = 1: $x' = \left\lfloor \frac{x}{q} \right\rfloor = 37$
3. s = 0: $x' = \left\lceil \frac{x+1}{1-q} \right\rceil = 87$

Binary ANS coder - example

4. s = 1: $x' = \left\lfloor \frac{x}{q} \right\rfloor = 154$
5. s = 0: $x' = \left\lceil \frac{x+1}{1-q} \right\rceil = 355$
6. s = 0: $x' = \left\lceil \frac{x+1}{1-q} \right\rceil = 814$
7. s = 0: $x' = \left\lceil \frac{x+1}{1-q} \right\rceil = 1863$
8. s = 1: $x' = \left\lfloor \frac{x}{q} \right\rfloor = 3312$

Binary ANS coder - example

9. s = 0: $x' = \left\lceil \frac{x+1}{1-q} \right\rceil = 7573$

10. s = 1: $x' = \left\lfloor \frac{x}{q} \right\rfloor = 13463$

11. s = 1: $x' = \left\lfloor \frac{x}{q} \right\rfloor = 23934$

12. s = 1: $x' = \left\lfloor \frac{x}{q} \right\rfloor = 42549$

13. s = 1: $x' = \left\lfloor \frac{x}{q} \right\rfloor = 75642$

Binary ANS coder - example

14.s = 0: $x' = \left\lceil \frac{x+1}{1-q} \right\rceil = 172899$

15.s = 1: $x' = \left\lfloor \frac{x}{q} \right\rfloor = 307376$

16.s = 0: $x' = \left\lceil \frac{x+1}{1-q} \right\rceil = 702576$

Interval limitation

- In long sequences, the number of bits required to represent encoder state will exceed the register capacity
- The state is limited to an interval of form $x \in [l, bl - 1], \quad l = 2^m, \quad b = 2^n$
- In case of overflow, output the lower *n* bits and update state to $x' = \frac{x}{b}$

ANS coding of larger alphabets

- Cannot perform direct calculations -> preprocessing is needed
- Preprocessing: tabulate all possible remainders
- Any reversible coding function that uniquely maps a remainder to a change of state is a valid ANS coding function

Preprocessing

- Sequence: 1, 3, 2, 3, 4, 2, 1, 4, 3, 3, 1, 4, 3, 1, 3, 1
- Frequencies: 5, 2, 6, 3
- For each symbol, generate *f(s)* pairs of type (symbol, index) :
  - (1, 0), (1, 1), (1, 2), (1, 3), (1, 4)
  - (2, 0), (2, 1)
  - (3, 0), (3, 1), (3, 2), (3, 3), (3, 4), (3, 5)
  - (4, 0), (4, 1), (4, 2)

Preprocessing

- Concatenate generated pairs into a 1D array
- Create a hash table
  - Key: pair (symbol, index)
  - Value: array index

Encoding

- Lookup array index *i* using (s, $x \bmod f(s)$)
- Calculate new state: $x' = \left\lfloor \frac{Nx}{f(s)} \right\rfloor + i$
- Tabulated pairs do not have to be used exactly once!

Decoder requirements

- Distribution of symbols
- Final state of the encoder
- Extra bits, output at renormalizations

ANS and JPEG-XL

- The coding standard JPEG-XL uses ANS coding for lossless entropy coding of metadata
- Limitations:
  - N = 4096
  - Number of symbols: 256
  - $x \in [2^{16}, 2^{32} - 1]$
  - Initial state: 0x130000

References

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