Radial basis functions

interpolation and approximation

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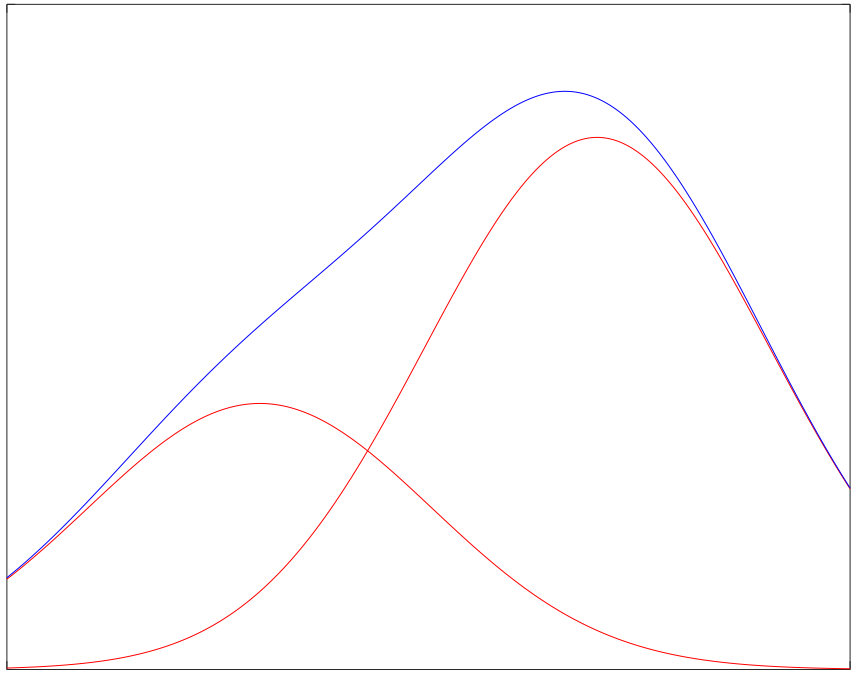
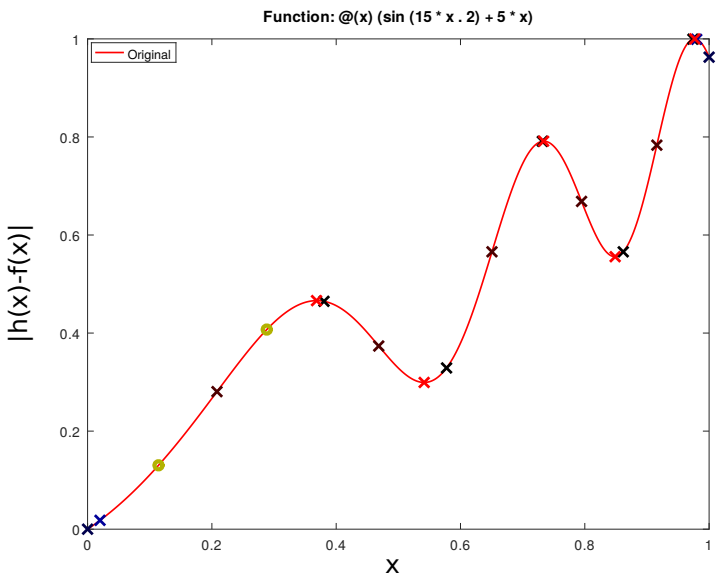
Introduction

- n-D interpolation and approximation technique
- Estimates function value by a sum of radial functions
- Example: sum of two red RBFs interpolates blue curve



Usage, 1D example

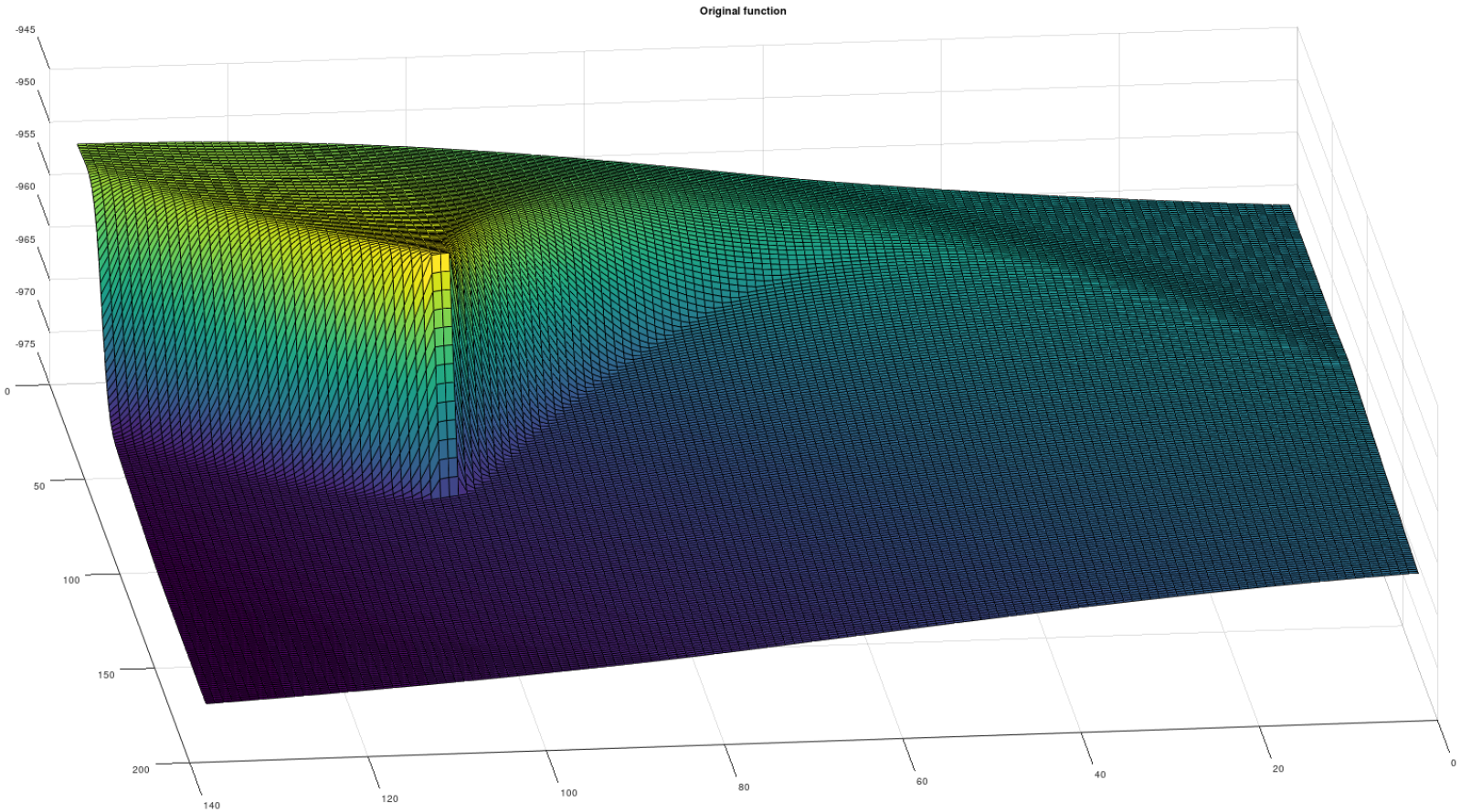
- Usage:
  - 1D signals
  - 2D images
  - 3D shapes
- Mainly 1D signals and 2D functions in my existing research
- Generalisation into higher dimensions possible



- Function: $\sin 15x^2 + 5x$
- 1000 uniform samples
- 21 RBF centres (compression approx. 1:23)
- Centres: extrema, inflexion points, on border, ...

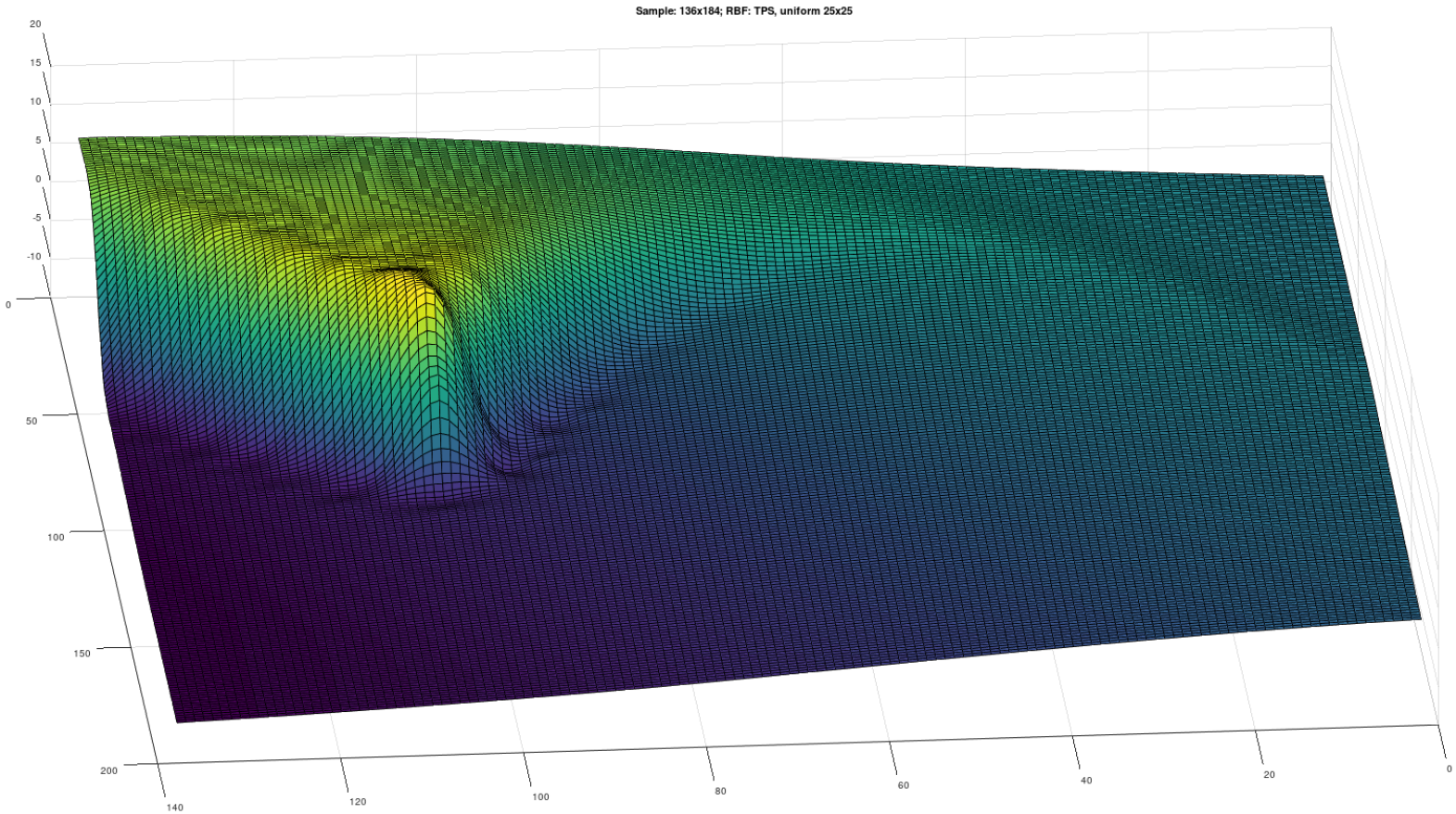
2D example 1/3

Original data: 136x184 grid



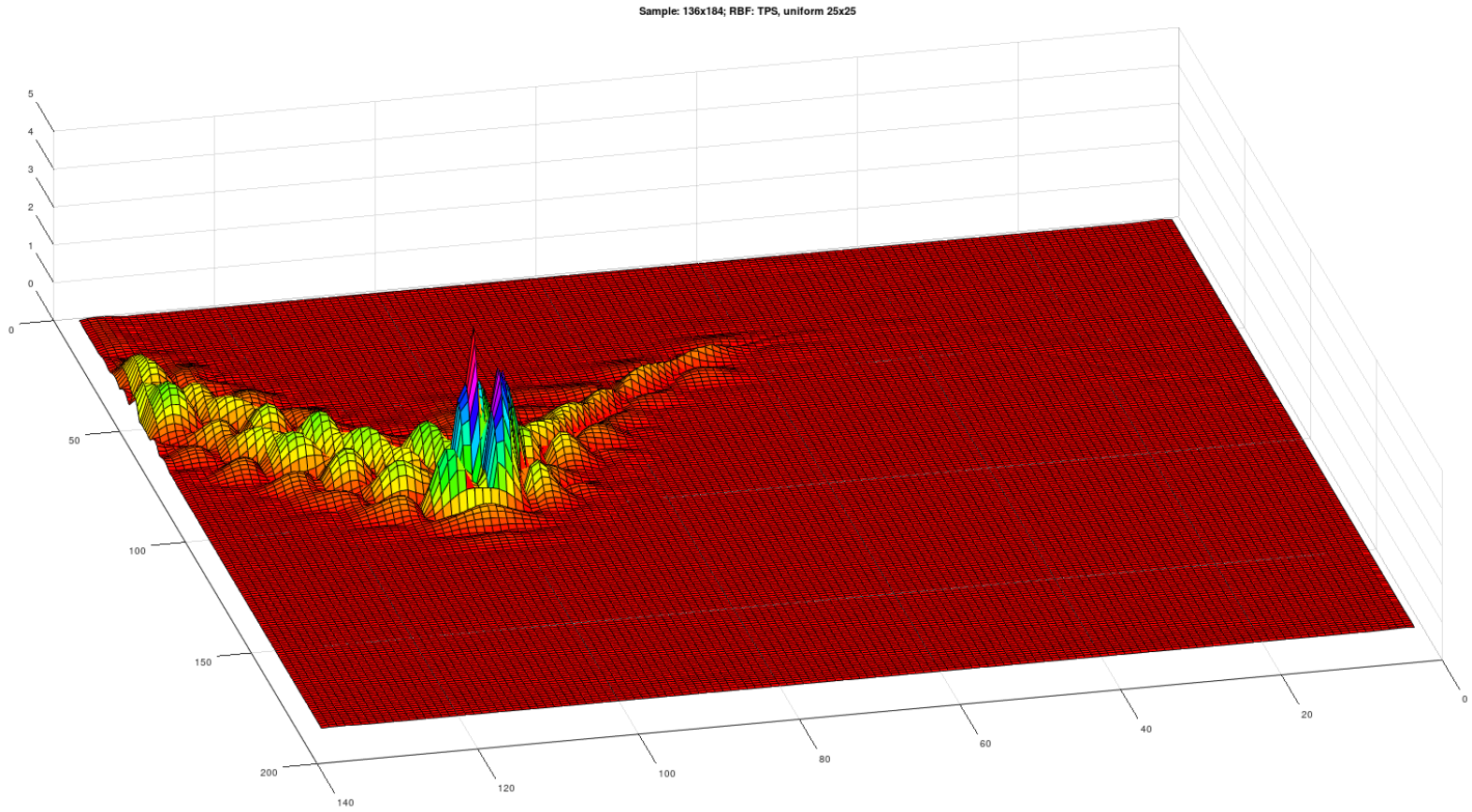
2D example 2/3

Thin plate spline, 25x25 uniform samples (approx. 1:13)

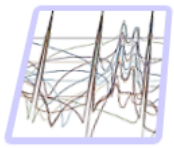


2D example 3/3

Error rates (abs. difference), max. relative error: 13%



Publications



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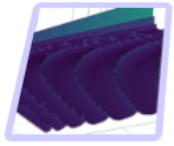
IEEE 18th World Symposium on Applied Machine Intelligence and Informatics, SAMI 2020,
pp.135-140, ISBN 978-1-7281-314, Slovakia, (2020) (Scopus)
UT WoS: 000589772600026, EID: 2-s2.0-85087093548, OBD: 43929006
https://doi.org/10.1109/SAMI48414.2020.9108712
[PDF]



Cervenka, M., Skala, V.:
Conditionality Analysis of the Radial Basis Function Matrix,
ICCSA 2020 proceedings, part II, LNCS, pp. 30-43,
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https://doi.org/10.1007/978-3-030-58802-1_3
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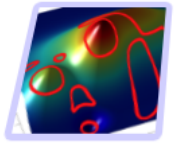

Cervenka, M., Smolik, M., Skala, V.:
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Computational Science and Its Application, ICSSA 2019 proceedings,
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UT WoS: 000661318700024, EID: 2-s2.0-85069157052, OBD: 43926678
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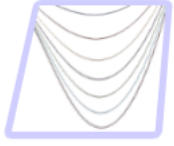

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Informatics 2019, IEEE proceedings,
pp.357-362, ISBN 978-1-7281-3178-8, Poprad, Slovakia, (2019)
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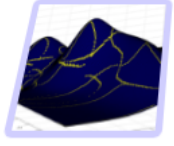

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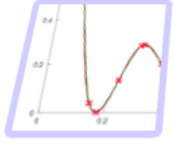

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Finding Points of Importance for Radial Basis Function Approximation of Large Scattered Data,
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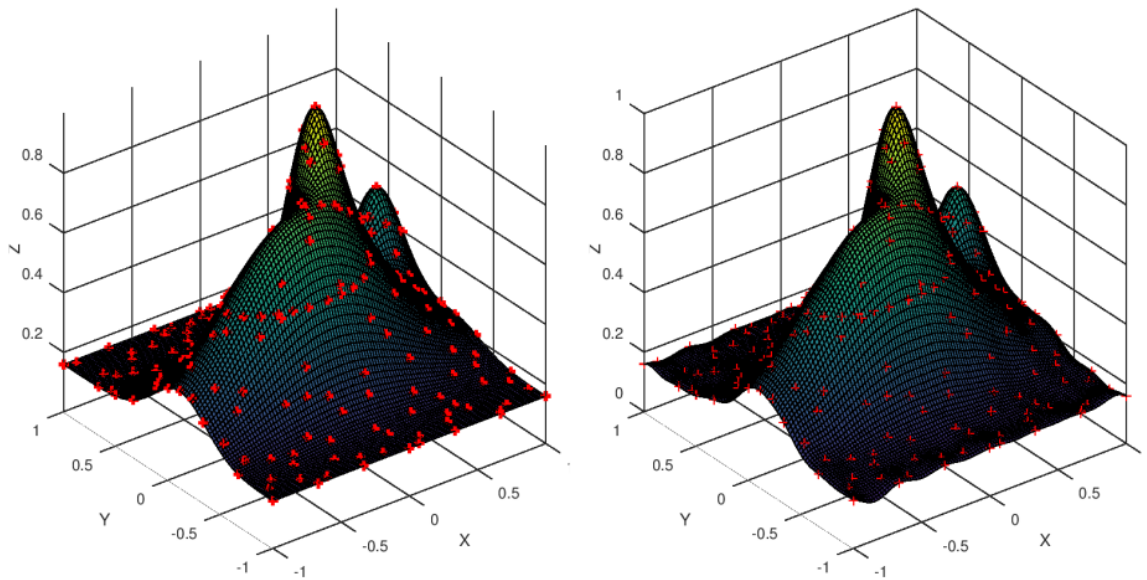
Publication 1

Cervenka, M., Smolik, M., Skala, V.: **A New Strategy for Scattered Data Approximation Using Radial Basis Functions Representing Points of Inflection**, Computational Science and Its Application, ICSSA 2019 proceedings, Part I, LNCS 11619, pp.322-226, ISSN 0302-9743, ISBN 978-3-030-24288-6, Springer, (2019)



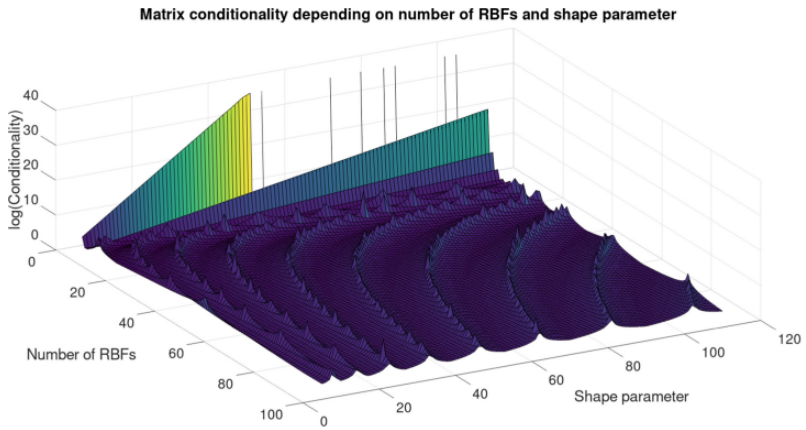
(a) Original data set.

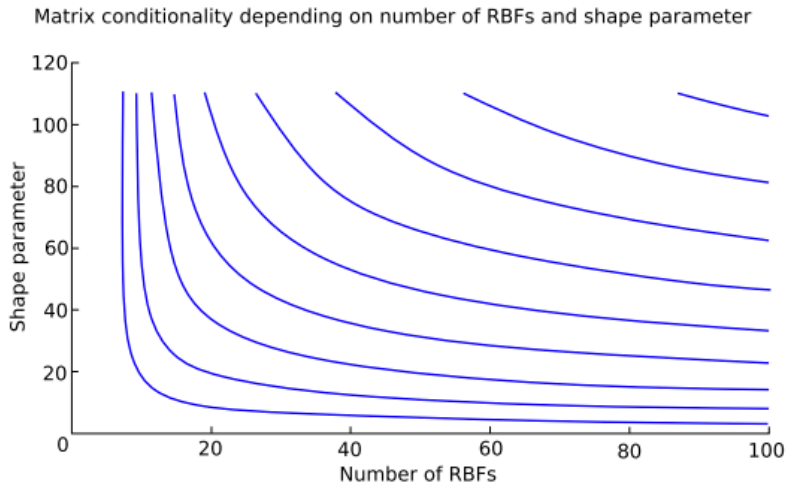
(b) RBF approximation.

Fig. 9. The RBF approximation of $2\frac{1}{2}D$ function (14). The total number of RBF centers is 244 (red marks).

Publication 2

Cervenka, M., Skala, V.: **Conditionality Analysis of the Radial Basis Function Matrix**, ICCSA 2020 proceedings, part II, LNCS, pp. 30-43, Springer, (2020)





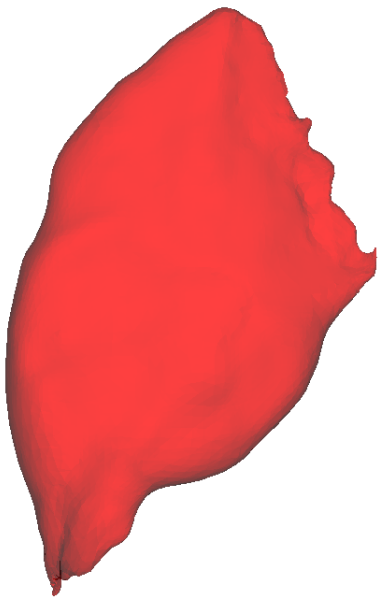
Combinations of RBFs and shape parameters, where the solution will be stable (valleys) and unstable (hills), using uniform sampling and Gaussian RBF.

Summary

+ Simple generalisation to higher dimensions
+ Can reach high data reduction ratio
+ Automatic smoothing property
+ Variety of RBFs to choose from

- Cannot reconstruct sharp edges
- Problems at the boundaries
- Equation system conditionality problems
- Finding suitable centre points (shape parameters)

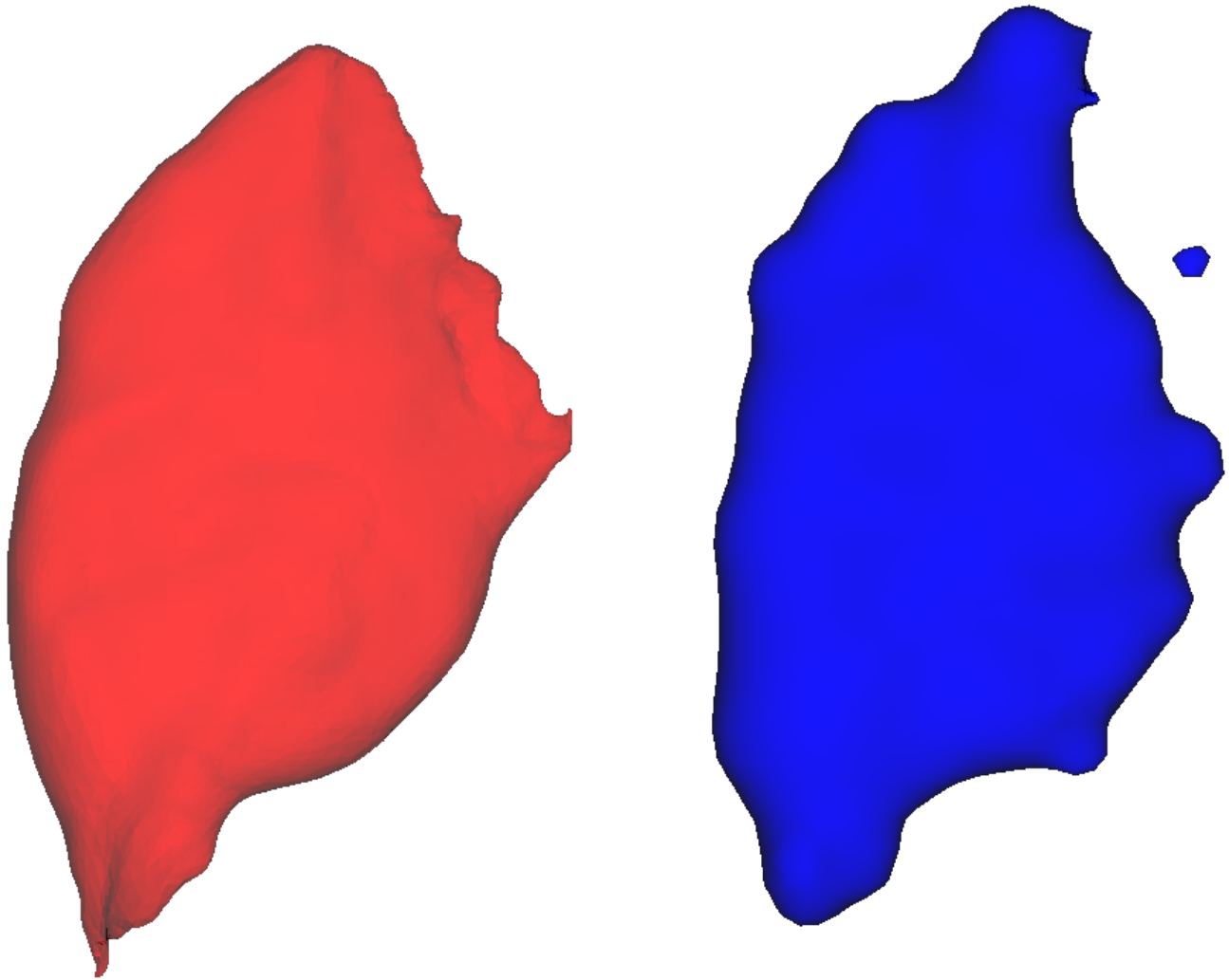
Homework (1/6)



- Gluteus maximus muscle
- Triangular mesh $\rightarrow$ RBF representation
- Data reduction (from 9878 vertices + connectivity)
- Features: centre points coordinates + shape parameters
- Residuals: differences between mesh and isosufrace (WIP)

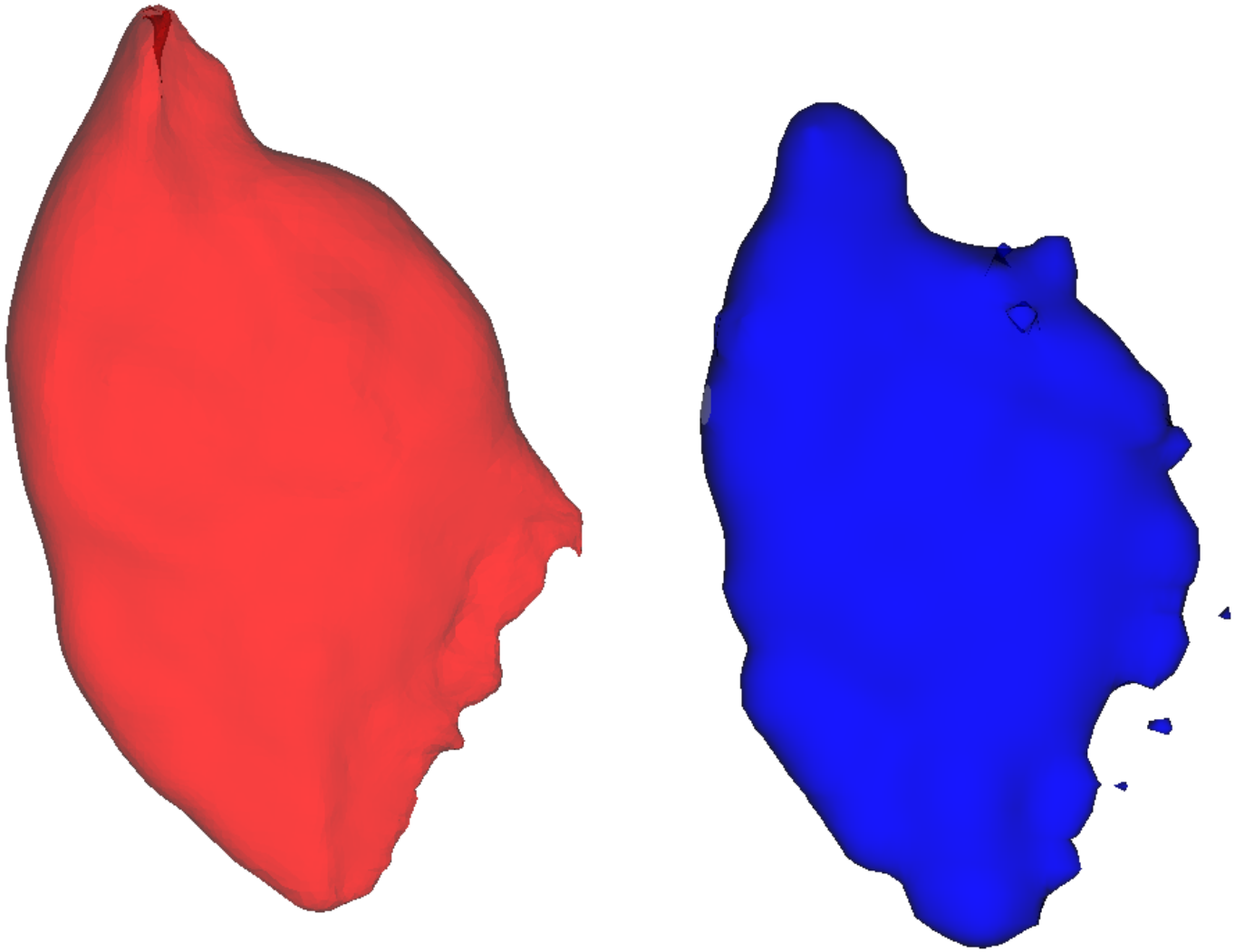
Homework (2/6)

50 RBFs (200 parameters), Jaccard index: 94.651% (100000 samples)



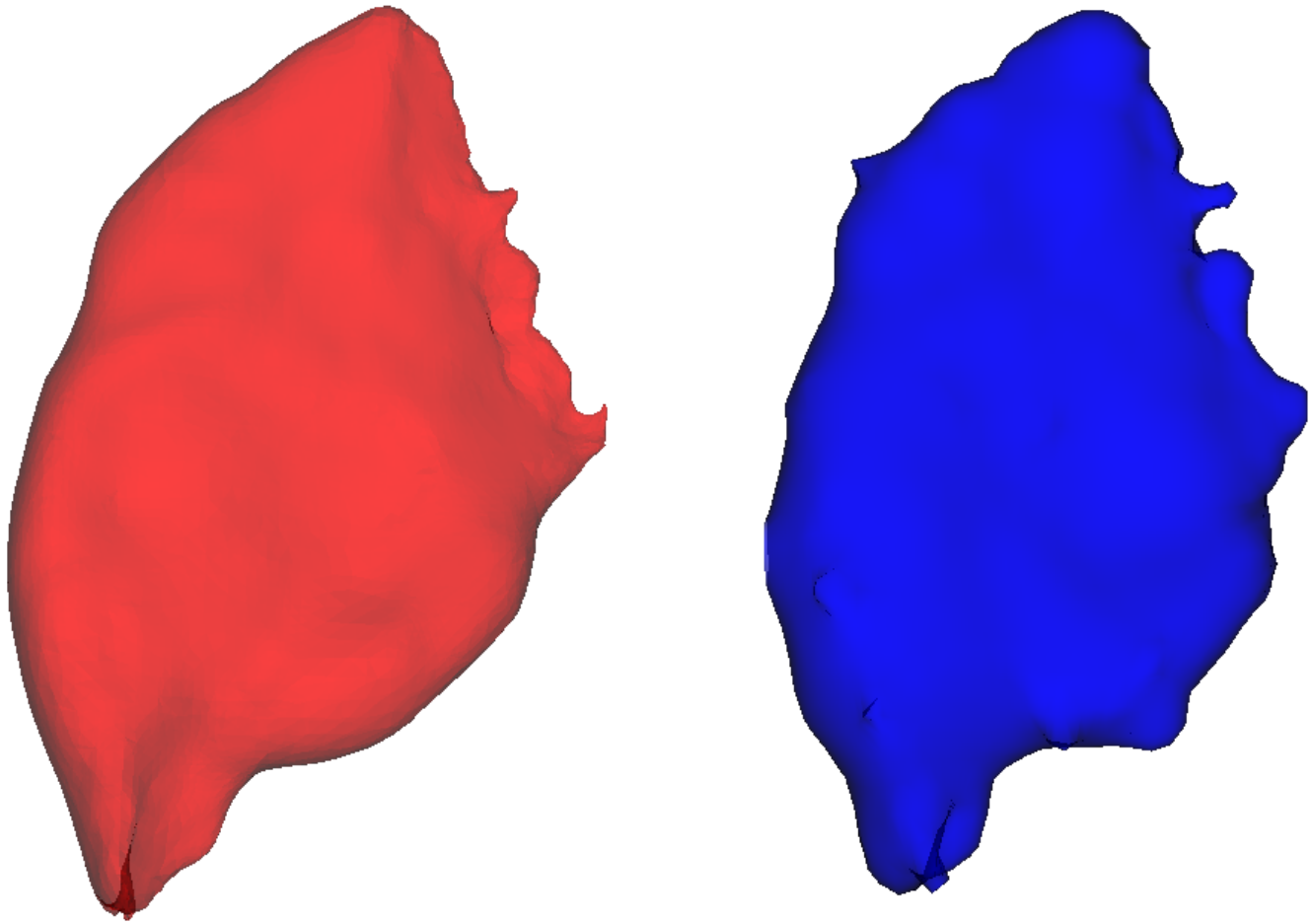
Homework (3/6)

100 RBFs (400 parameters), Jaccard index: 95.95% (100000 samples)



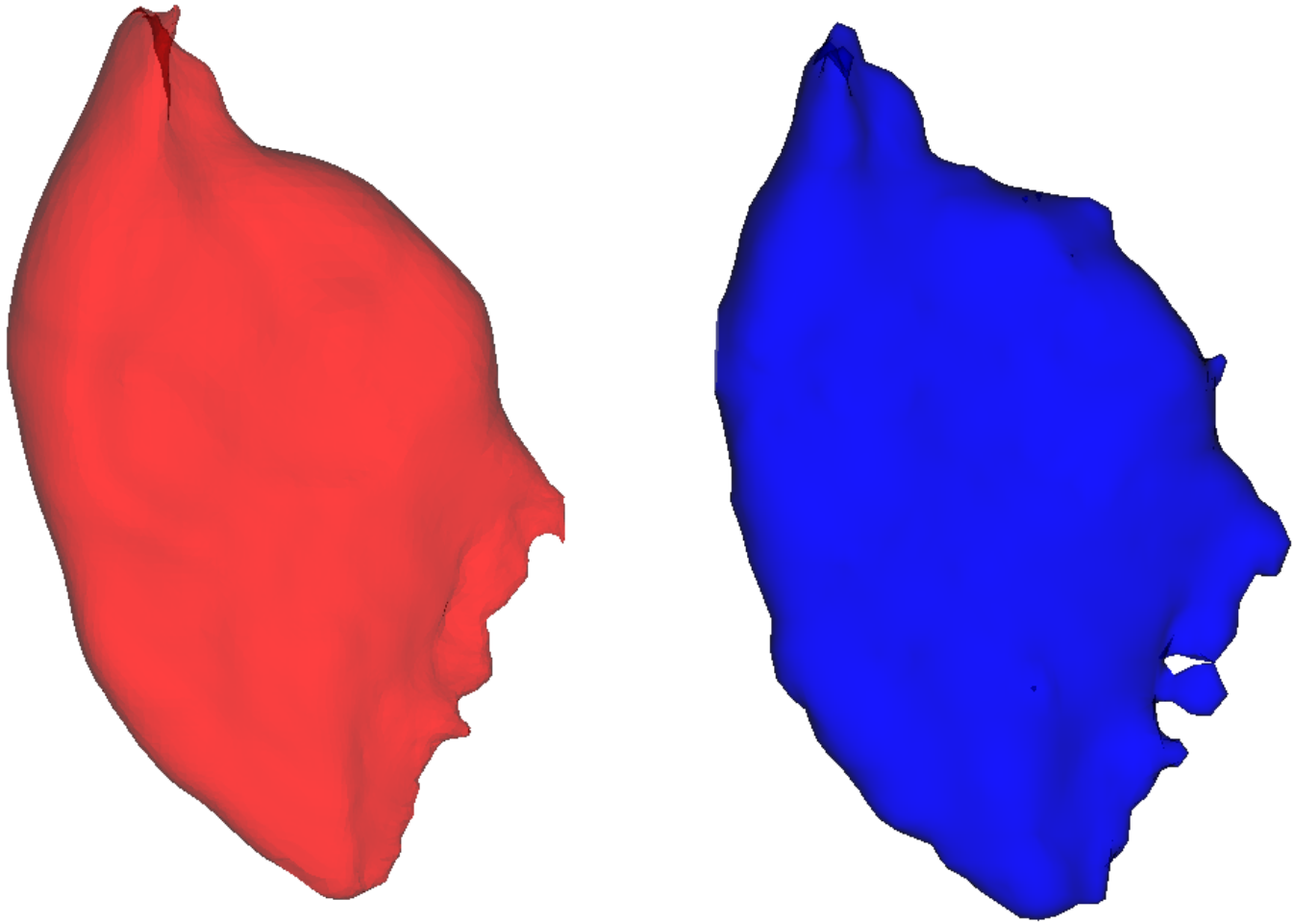
Homework (4/6)

200 RBFs (800 parameters), Jaccard index: 97.28% (100000 samples)



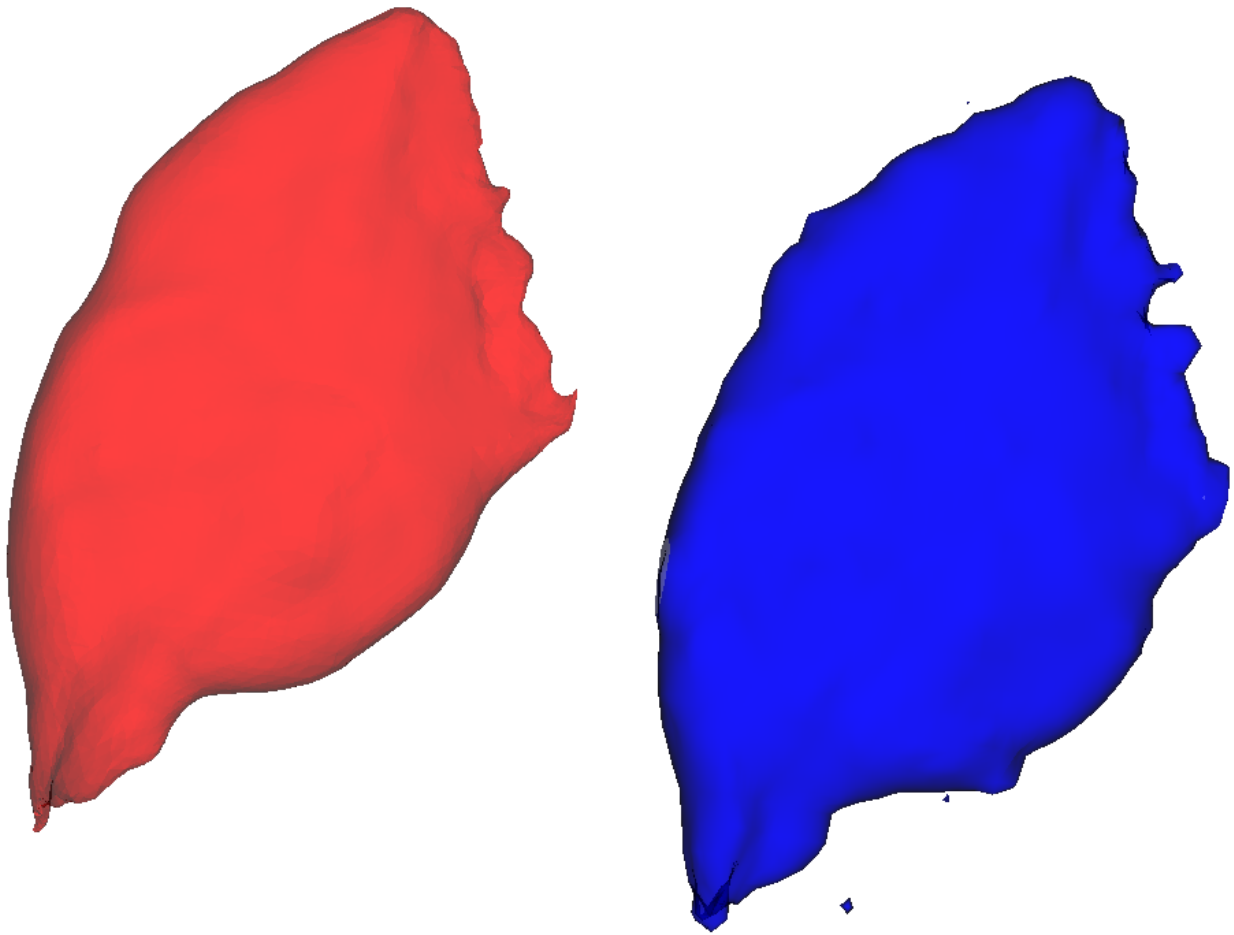
Homework (5/6)

500 RBFs (2000 parameters), Jaccard index: 97.945% (100000 samples)



Homework (6/6)

1000 RBFs (4000 parameters), Jaccard index: 98.783% (100000 samples)



Thank you for your attention