# Reflection symmetry detection in Earth observation data 



## Introduction

- Symmetry
- Transformation sym which transforms a set $S$ into itself.
- Informally, symmetric set (e.g. points belong to symmetry).
- Reflection (reflectional, mirror) symmetry
- $a \in S \Rightarrow \operatorname{sym}(\operatorname{sym}(a))=a$
- In Euclidean space, the following symmetries fulfil this condition (necessary, but not sufficient for our purpose):
- Equivalence: $a \in S \Rightarrow \operatorname{sym}(a)=a$
- Rotation for $180^{\circ}$ around point in 2D or around arbitrary curve in 3D.
- Point (central) symmetry (in 2D the same as rotation for $180^{\circ}$ ).
- Reflection across arbitrary curve in 2D or arbitrary surface in 3D.
- Our goal: reflection across line in 2D or across plane in 3D.


## Introduction

- Let In represent the input set for symmety detection.
- Global symmetry: $S=I n$.
- Partial symmetry: $S \subseteq \ln$.
- Local symmetry: $S \subseteq I n$ and $S$ is connected.
- Different informal definitions (with different meaning) possible, e.g. that S should contain all the points in some area around the axis, in bounding box, convex or concave shell.
- Each global symmetry is also partial.
- Each local symmetry is also partial.
- As the partial symmetry is a super-class for the local and global ones, our algorithm detects partial symmetries and then, if required, checks the conditions for the local and global symmetry.


## Earth observation (EO) data

## - Sampled data

- Points of an „original"and mirrored part rarely match exactly $\rightarrow$ approximate symmetry detection (based on voxelization).
- Top view
- 3D data, (mostly) acquired „down" from a satellite, airplane...
- More data collected from visible top faces than from side and bottom faces $\rightarrow$ more likely to find symmetries on top faces.
- Usually, width and length of the considered area are much greater than the range of altitudes.
- Only vertical symmetry planes considered!!!
- Symmetries found in horizontal voxel slices and then merged.


## Concept of the new method

－Maximal symmetries only．
－Strength of symmetry＝amount of involved input data （｜S｜／｜In｜or expressed with number of voxels）．
－Bottom－up approach：
－Basic symmetries found in each horizontal slice，followed by intra－ slice and inter－slice merging of symmetries．
－Basic symmetry：
－Between two line segments（LSs），．．．
－Non－material，material，and interesting voxels
－Line segment：
－A pair of interesting voxels（endpoints）．In the same slice．
－Length．Threshold（80 \％）interesting voxels．
－Each LS in（basic）symmetry has its symmetrical pair with the same lenght somewhere in the considered slice．

## Partial symmetry

Voxelization.
Identify the material voxels (and filter them).
Identify the interesting voxels.
For each horizontal slice of the voxel grid
Identify LSs and cluster them with respect to their lengths.
For each cluster of LSs
Establish basic symmetries among pairs of LSs.
Merge symmetries.
Merge symmetries from different clusters.
Merge symmetries from different slices.
Add „non-interesting" material voxels into symmetries.
Extend $S$ by mirroring its voxels accurately.
Postprocessing.

## Latest ilmprovements

- Extending S by mirroring its voxels accurately.
- Due to approximate geometry, symmetric pairs of a voxel centre and of a point in the same voxel might be different.
- For each voxel in $S$, add all its symmetric pairs into $S$.
- Postprocessing
- Small groups of voxels may be filtered out from $S$.
- Symmetries with similar symmetry planes may be merged.
- Sort the detected symmetries according to the similarity with a chosen symmetry axis/plane.
- Separately detect symmetries in individual layers (buildings, trees, ground...) and then merge them. Usually, It makes no sense that a voxel on a tree and a voxel on a roof are treated as symmetric.


## Global and local symmetry

- Global symmetry
- Common symmetry plane in all non-empty slices
- 100\% involvement of input data in $S$ in all slices
- More global or partial symmetries may be detected (with different planes of symmetry).
- Local symmetry
- Obtained by the decomposition of the partial/global symmetry
- Each connected component intersected by the symmetry plane represents the local symmetry.
- Green voxels + adjacent red and blue voxels.


## Results




GeoSym


GeoSym

## Grad castle (Goričko, Slovenia)

, 273540 points (buildings + trees).

- Left: 1 of 29.831 partial symmetries. 1 m voxels, 1.629.909 voxels, 52.141 interesting voxels, 14.973 s (+268 s for local symmetries).
- Right: 1 of 4438 partial symmetries. 5 m voxels, 62196 voxels, 3721 interesting voxels, 182 s (+9 s for local symmetries).



## Grad castle

- Left: 940 of 979 partial symmetries.
- Right: 1876 of 8616 local symmetries.



## Ljudski vrt（Maribor，Slovenia）

－ 126.714 points（buildings only）
－Left： 1 of 6863 partial symmetries， 98.496 voxels， 2579 interesting voxels， 23 s （＋2 s for local symmetries）．
－Right： 244 of 24.404 local symmetries．


## Slomšek square (Maribor, Slovenia)

- 86.107 points (buildings + trees)
- Left: 1 of 5082 partial symmetries, 42.480 voxels, 2478 interesting voxels, 35 s (+4 s for local symmetries).
- Right: 2465 of 5082 partial symmetries.



## Maribor cathedral

- 11779 points (building)
- 1st and 23th of 64 partial symmetries, 5415 voxels, 380 interesting voxels, 0.03 s (+0.01 s for local symmetries).


GeoSym

## Maribor cathedral

- Strongest symmetry at 500 voxels. 8 slices.



## Bled lake Island (Slovenia)

- 219.186 points (buildings + trees, without water)
- 1st and 23th of 979 partial symmetries, 9520 voxels, 538 interesting voxels, 2.38 s (+0.3 s for local symmetries).



## Punta Piran (Slovenia)

- 99.413 points (buildings only)
- 1st and 9352 nd of 12.177 partial symmetries, 25.326 voxels, 1699 interesting voxels, 44 s (+6 s for local symmetries).
- 1st and 14.055 th of 18.556 local symmetries



## Conclusion

- Time complexity $O\left(n^{4}\right)$, where $n$ is the number of voxels.
- Reduction to interesting voxels, clustering due to length of LSs, and separate consideration in slices are crucial accelerations.
- Satellite images may also be considered (e.g. greyscales treated as altitudes).
- The partial symmetry part has been presented at the international conference MATCOS-22 in Koper, Slovenia, in the first half of October 2022.
- Journal paper will be submitted to MDPI Sensors in few days.
- Partial rotational symmetry detection was implemented on mostly the same principles (to be published after some other priorities).

